

GEO1003 - Shared Notes

Master Geomatics Students

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1 Introduction

These notes were created by students from the MSc Geomatics for the Built Environment at TU Delft.

They are based on the lectures and literature provided by the course GEO1003 (Positioning and Location Awareness) by Edward Verbree.

2 How does GNSS work?

2.1 Introduction

GPS (Global Positioning System), also known as **NAVSTAR** (NAVigation Satellite Time And Ranging) had its first satellite launched in 1978.

Below are the 4 constellations of GNSS systems:

- **Global Positioning System (GPS)** - United States
 - *Standard Positioning Service (SPS)*
 - *Precise Positioning Service (PPS)*
- **GLONASS** - Russia
- **Galileo** - Europe
 - *Open service (OS)*
 - *Public regulated service (PRS)*
- **Beidou(/Compass)** - China

Their properties are:

Table 8.14 Properties of GNSS Mid-Earth Orbits

<i>Constellation</i>	<i>Number of Planes</i>	<i>Radius (km)</i>	<i>Height (km)</i>	<i>Period</i>	<i>Orbits per Sidereal Day</i>	<i>Ground-Track Repeat Period (Sidereal Days)</i>	<i>Inclination Angle</i>
GPS	6	26,580	20,180	11 hr, 58 min	2	1	55°
GLONASS	3	25,500	19,100	11 hr, 15 min	2.125	8	64.8°
Galileo	3	29,620	23,220	14 hr, 5 min	1.7	10	56°
Beidou	3	27,840	21,440	12 hr, 52 min	1.857	7	55°

Figure 1: Properties of GNSS systems

2.2 Fundamental Concepts and Overview

Notes from the chapter Introduction of the book “Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems” - Paul D. Groves

2.2.1 Fundamental concepts

Navigation refers to determining and planning the movement of a body, such as a ship or aircraft, by calculating its position and course. It involves both the science of positioning and the planning of a course to avoid obstacles.

Positioning is a subset of navigation, involving the determination of a body's position. Techniques can be classified based on:

1. **Real-time vs. postprocessed techniques**
2. **Fixed vs. movable objects** (static vs. mobile positioning)
3. **Self-positioning vs. remote positioning**

Navigation systems use various sensors, such as accelerometers and gyroscopes, to calculate position and velocity. The output is a **navigation solution**, typically including position and velocity, and sometimes attitude and acceleration. Systems may calculate 2D or 3D positions based on the application (e.g., cars vs. airplanes).

The **user** in navigation refers to the person or software that receives the position and velocity solution, and **user equipment** refers to the system located on the object being positioned.

Position and location are related but distinct concepts:

- **Position** is quantitative (coordinates).
- **Location** is qualitative (e.g., a city or room).

Localization is used instead of positioning particularly for short-range applications.

Two fundamental methods for determining position:

- **Position Fixing:** Uses identifiable external information such as radio signals, acoustic, ultrasounds etc or environmental features (signs, roads, terrain, landmarks, sounds smells).
- **Dead Reckoning:** Measures distance and direction traveled using a self-contained system, like an INS, without relying on external infrastructure. Environmental features can also assist dead reckoning by comparing repeated measurements.

2.2.2 Dead Reckoning

Calculates current position by integrating changes in position or velocity from a previous location, requiring attitude information for accurate direction.

- 2D Navigation: Only a heading measurement is needed.
- 3D Navigation: Requires a full attitude measurement in three components.
- Accuracy: Smaller calculation steps enhance accuracy, particularly with changing attitudes.

Modern Methods: Include pedometers and advanced pedestrian dead reckoning using accelerometers. Odometer: Measures distance through wheel rotations, used in vehicles and marine/aircraft equivalents. Heading Measurement: Done using magnetic compasses, gyrocompasses, Land heading, roll and pitch (measured using accelerometers, tilt sensors, or celestial observations (sun, moon, stars)), and Gyroscopes and differential odometry. Inertial Navigation Systems (INS): Use accelerometers, gyroscopes, and a navigation processor to calculate position, velocity, and attitude.

Performance:

- Navigation accuracy depends on sensor quality
- The principal advantages of inertial navigation and other dead-reckoning techniques, compared to position fixing, are continuous operation, a high update rate, low short-term noise, and the provision of attitude, angular rate, and acceleration as well as position and velocity.
- The main drawbacks are that the position solution must be initialized and the position error grows with time because the errors in successive distance and direction measurements accumulate.
- In an integrated navigation system, position-fixing measurements may be used to correct the dead-reckoning navigation solution and also calibrate the dead-reckoning sensor errors.

2.2.3 Position Fixing

2.2.3.1 Position-Fixing Methods There are five main methods: proximity, ranging, angular positioning, pattern matching, and Doppler positioning.

- **Proximity:**
Simplest method, assumes the receiver's position is the transmitter's or a nearby feature's. Accuracy improves with proximity to landmarks. Short-range signals like Bluetooth, RFID, or indoor features are ideal. Using multiple landmarks allows averaging of their positions.
- **Containment Intersection:** A refined proximity method where containment zones around landmarks are defined. Observed landmarks localize the position to the intersection of these zones, with the center of the intersection as the position fix.
- **Ranging:**
Measures distances to landmarks and defines circular lines of position (LOPs) based on these ranges. The user's position lies at the intersection of LOPs. At least two LOPs are needed; three resolve ambiguities. (Or prior information)
In three-dimensional positioning, each range measurement defines a spherical surface of position (SOP) centered at a landmark. The intersection of:
 - Two SOPs forms a circular line of position (LOP).
 - Three SOPs intersects at two points, requiring additional information or a fourth SOP to determine a unique position fix.

If the user and landmarks are coplanar, only planar position components can be determined, limiting vertical accuracy in terrestrial systems.

Range Measurement: based on signal time of flight (TOF), multiplied by the speed of light or sound. Accurate TOF measurement depends on transmitter-receiver time synchronization.

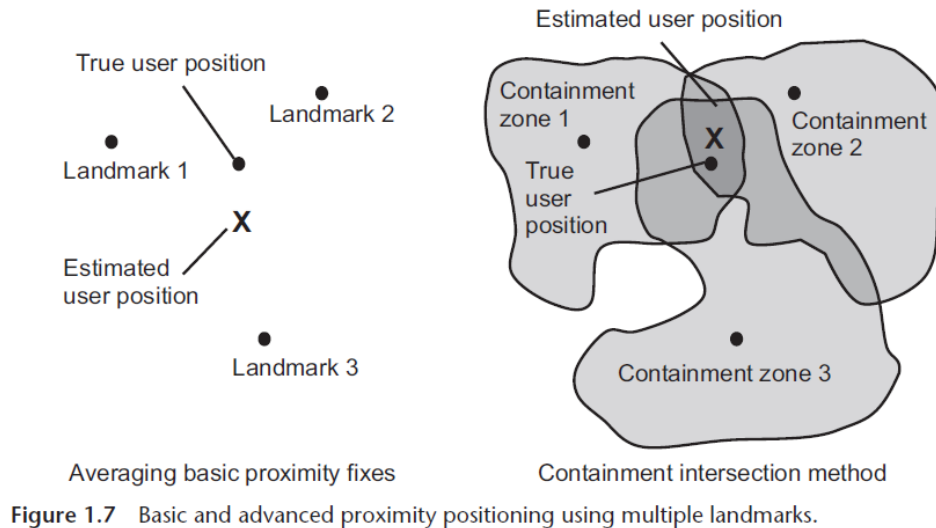


Figure 2: Proximity positioning

- Two-way ranging (e.g., distance measuring equipment - DME) minimizes synchronization errors via bidirectional signal exchange.
- One-way ranging (e.g., GNSS) requires synchronized transmitter clocks, treating receiver clock offset as an unknown. This technique is known as passive ranging and is how GNSS works. Additional transmitters or reference receivers can correct for clock offsets.
- Where the landmark is an environmental feature, active sensors like radar, sonar, or laser transmit signals to a landmark, measure the reflected signal's round-trip time, and calculate the range.

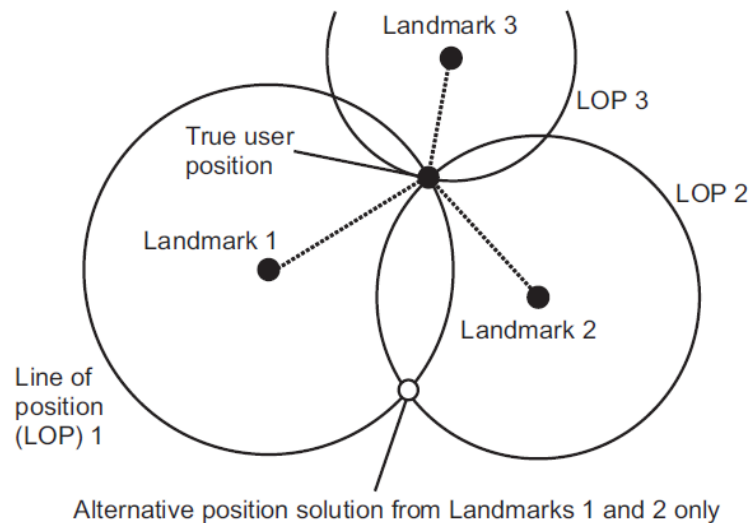


Figure 1.8 Positioning by ranging in two dimensions.

Figure 3: Positioning by ranging

- **Bearing:**

A bearing is the angle in the horizontal plane between the line of sight to an object and a reference direction (e.g., true or magnetic north).

Position Fixing with Bearings:

- Two-Dimensional Positioning: Measuring the bearing to two known landmarks defines straight LOPs. Their intersection gives the user's position. Without a reference direction, measuring the angle difference between two landmarks defines a curved LOP, requiring three landmarks for positioning.
- Three-Dimensional Positioning: Extends by measuring the elevation angle (line of sight vs. horizontal plane) to one landmark. However, accuracy decreases with distance, and Earth's curvature affects measurements.

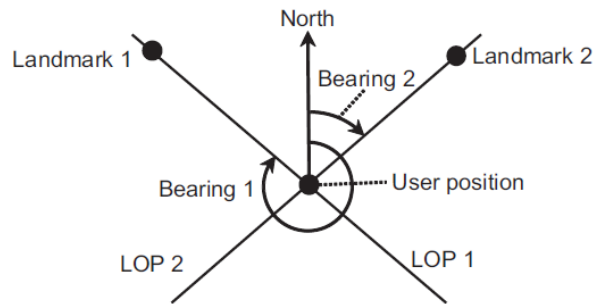


Figure 1.9 Angular positioning in two dimensions.

Figure 4: Angular positioning

- **Angle of Arrival (AOA):**

Direction Finding: Uses steerable directional antennas to determine signal bearing.

- Nonisotropic Transmissions: Broadcast signals with modulation varying by direction enable bearing and/or elevation determination. Examples include VHF omnidirectional radiorange (VOR) and Nokia HAIP (high-accuracy indoor positioning).
- Environmental features can be measured using cameras, laser scanners, imaging radar, or multibeam sonar. The position of a feature in the sensor's image, combined with sensor orientation, determines its bearing and elevation.

- **Integrated Navigation System:**

Full position fixes are unnecessary; single measurements (e.g., range, bearing, or elevation of a landmark) can contribute to the navigation solution. For example, a two-dimensional position fix may be obtained by measuring the range and bearing of a single landmark as shown in Figure 1.10. Adding elevation provides a three-dimensional fix.

- **Landmark Identification:**

Signals are identified via demodulation, transmitter IDs, or frequency. Environmental features are matched with stored data, requiring distinctive features and approximate position inputs to limit the size of database that must be searched to obtain a match. Even so, positioning using environmental features is normally more processor intensive than signal-based positioning.

- **Pattern Matching:**

A database contains parameters varying by location (e.g., terrain height, signal strength, magnetic fields, GNSS obstructions). Measured values at the user's position are compared with database values at candidate grid points. The best match determines the position. Interpolation refines results if several candidates match well. As with feature matching for landmark identification, the input of an approximate position solution limits the size of the database to be searched.

Combining multiple parameters into a **location signature** enhances uniqueness and accuracy. In some cases, such as terrain height, there is insufficient information to obtain an unambiguous position fix. However, if the navigation system is moving, measurements may be made at multiple positions, collectively known as a transect. Position fixing can be enhanced by this. Using dead reckoning to relate transect points allows measurements to form a location signature, compared with a database for positioning, requiring interpolation if spacing differs.

- **Doppler Positioning:**

Measures Doppler shift from relative transmitter-receiver motion to determine relative velocity and a conical surface of position (e.g., used in Iridium positioning).

- **Height Measurement:**

Barometric Altimeter: Uses pressure to estimate height or underwater depth. Radar Altimeter: Measures height above terrain for aircraft where terrain height is known.

- **Data and Databases:**

Position-fixing methods rely on data like landmark positions, feature info, and pattern-matching parameters. Databases may be preloaded but require updates and significant storage for large areas.

- **SLAM (simultaneous localization and mapping):**

Allows systems to create and update its own landmark databases by exploring environments, observing features several times and using dead reckoning to track traveled distances.

Many signal-based positioning systems include transmitter positions in their signals, but this can delay position computation after signal reception. A separate data link, known as assistance, can provide the necessary information on demand to reduce delays.

Position fixing is essential for determining absolute position, with errors independent of the distance traveled, but it depends on the availability of suitable signals or environmental features. To enhance position accuracy and availability, multiple position-fixing technologies or dead reckoning can be combined to bridge gaps.

2.2.3.2 Signal-Based Positioning Signal-Based Positioning covers various navigation systems, from early radio-based methods to modern satellite and terrestrial systems.

- **Early Radio Navigation:** Radio navigation began in the 1920s, with Omega providing global coverage by the 1970s.
- **Terrestrial Systems:** Older systems like DME, VOR, and Loran provide long-range navigation, while newer methods use signals from mobile phones, Wi-Fi, Bluetooth, RFID, UWB, television, and broadcast radio for short-range positioning.
- **Signal of Opportunity (SOOP):** Some systems use signals like mobile phone or broadcast signals without operator cooperation, relying on calibration or reference stations.
- **Underwater Navigation:** Acoustic signals, such as ultrasound, are used for short-range underwater positioning.
- **Satellite Navigation:** GNSS systems (e.g., GPS, GLONASS, Galileo) provide global positioning, requiring at least four satellites for 3D fixes. GNSS offers high accuracy but is vulnerable to interference, jamming, and obstructions. Differential and carrier-phase techniques enhance accuracy, achieving meter to centimeter-level precision.
- **GNSS vs Terrestrial Systems:** GNSS offers global coverage and better accuracy, but is vulnerable to interference. Terrestrial systems like DME and enhanced Loran serve as backups, with short-range systems covering areas where GNSS signals are weak.
- **Maximizing Positioning:** Combining different signal types improves the robustness and availability of navigation solutions.

Overall, GNSS provides superior accuracy and global reach, while terrestrial and short-range systems fill gaps in environments where GNSS struggles.

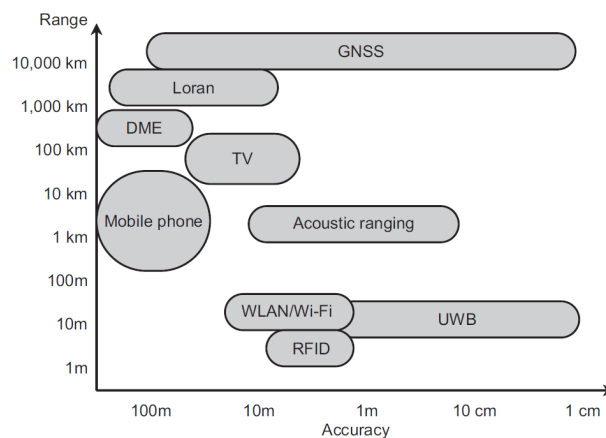


Figure 1.13 Range and accuracy of signal-based positioning technologies.

Figure 5: Range and accuracy

2.2.3.3 Environmental Feature Matching

- **Natural Navigation:** Humans and animals navigate using environmental features, often comparing them with maps, memory, or written directions.
- **Static or Predictable Features:** Features used for navigation must be either stationary or predictable in their movement.
- **Historical Positioning:** Historically, position fixes (LOPs) were made by manually identifying distant landmarks and using tools like a theodolite and compass.
- **Modern Techniques:** Image-based positioning techniques now automate the process, and devices like stereo cameras, radar, laser scanners, or sonar enable both angular positioning and ranging.
- **Pattern Matching:** Position can also be inferred directly from images through pattern-matching techniques.

Celestial Navigation:

- Using Sun, Moon, and Stars: Celestial bodies like the sun and stars have historically served as landmarks. For example, the sun's highest angle at the equinox equals the latitude.
- Star Navigation: The position can be determined by measuring the elevation angles of stars, and accurate time is required to calculate longitude. This has been practical since the 1760s, after advances in timekeeping by John Harrison.
- Modern Tools: Today, star imagers automate this celestial navigation process.

Terrain-Referenced Navigation (TRN):

- User Position from Terrain Height: TRN determines position based on the terrain height below the vehicle.
- Aircraft: Uses radar altimeters (radalt) or laser scanners to measure height above terrain, then compares it with the vehicle's height to determine the terrain height.
- Ship/Submarine: Sonar measures the depth of the terrain below.
- Land Vehicles: Inference of terrain height is made directly from their height solution.
- Pattern Matching: The vehicle's terrain measurements are compared with a terrain database to determine position.
- Accuracy and Limitations: Radalt-based navigation is accurate to about 50m, and works best over hilly or mountainous terrain. It is not effective over flat terrain.

Map-Matching Techniques:

- Constraints on Dead Reckoning: Land vehicles follow roads or rails, so map-matching uses this to constrain dead reckoning and correct positioning errors.
- Car Navigation: Map matching is crucial in car navigation, combining proximity and pattern-matching methods.
- Height Inference: Maps can also help infer terrain height from horizontal position.

Other Environmental Features:

- Magnetic/Gravity Anomalies & Pulsars: These can also be used for position fixing.
- Heterogeneous Feature Mix: A mix of different environmental features may be used to improve position determination.

Pattern Matching & Fault Detection:

- False Matches: Pattern matching may occasionally result in false or ambiguous position fixes.
- Fault Detection: It's important to implement fault detection and recovery techniques to address these potential errors.

2.2.4 The Navigation System

Diverse Needs Across Applications: Navigation requirements differ significantly depending on the application. Factors include accuracy, update rate, reliability, budget, size, and mass. Some applications also require attitude solutions in addition to position and velocity.

2.2.4.1 Requirements Navigation needs differ by application, with factors such as accuracy, reliability, and cost playing a role.

- High-Value Applications (e.g., airliners) need high integrity and reliability.
- Military systems must withstand electronic warfare and varying accuracy demands.
- Personal/Vehicle Navigation focuses on cost, size, and power consumption.
- Positioning systems may use custom infrastructure for high-value applications or general sensors for lower-value ones.

2.2.4.2 Context The system adapts based on the environment (e.g., land, sea, air) and behavior of the vehicle or user. Context-sensitive factors like physical constraints (e.g., vehicles following roads), speed limits, and environmental conditions influence navigation performance. Context-adaptive navigation allows systems to adjust dynamically, such as moving from indoor to outdoor environments.

2.2.4.3 Integration An integrated navigation system combines two or more navigation technologies, either position-fixing methods or a mix of position-fixing and dead reckoning.

Integration Types:

Measurement-Domain Integration:

Combines raw measurements (e.g. ranges, bearings) into a common position estimation algorithm. Advantages:

- Allows systems to contribute even without enough data for an independent solution.
- Easier error characterization ensures optimal weighting.

Position-Domain Integration:

- Inputs position solutions directly from different systems.

Typical Architecture:

- Combines position-fixing (e.g. GNSS) with dead-reckoning systems (e.g. INS).
- Dead-reckoning provides a continuous navigation solution, while position-fixing corrects errors and calibrates dead-reckoning sensors.
- Corrections and calibration are managed by an estimation algorithm, typically a Kalman filter.

Benefits:

- Exploits complementary error characteristics of position-fixing and dead reckoning.
- Enhances robustness and accuracy of the integrated solution.

2.2.4.4 Aiding Position-fixing and dead-reckoning systems can be aided using the integrated navigation solution or other positioning technologies.

Dead Reckoning Aiding:

- Requires initialization of position, velocity, and sometimes attitude.
- Integration algorithms provide periodic corrections to navigation solutions and sensor outputs.
- Estimated sensor errors can be fed back to improve accuracy.

Position-Fixing Aiding:

- Pattern-matching systems need an approximate position input to limit the search area and improve efficiency.
- Signal-based systems can use this approximate position to search for signals effectively.
- Tiered positioning: One system provides a coarse position, aiding another system in achieving higher precision.

Velocity Aiding:

- Used in transect-based techniques like Terrain-Referenced Navigation (TRN) to create a location signature by combining measurements from different positions.
- Enhances sensitivity in radio positioning systems and compensates for motion effects in two-way ranging systems.

2.2.4.5 Assistance and Cooperation Assistance:

- Utilizes a separate communication link to provide navigation systems with information about signals and environmental features for positioning.
- Includes data on transmitter locations, signal characteristics, landmark identification, and pattern-matching information.
- Benefits include access to up-to-date data, reduced onboard storage needs, faster position fixes, and support in areas with poor signal reception.
- Assistance data is often provided by commercial services, like mobile operators, with potential subscription fees.

Cooperative Positioning:

- Involves direct data exchange between nearby users over short-range links (e.g., **collaborative** or **peer-to-peer** positioning).
- Participants can include personnel, vehicle fleets, or public users, enabling cross-device cooperation (e.g., smartphone with a car or train).
- Shared data can include:
 - Signal quality and availability.
 - Transmitter clock offset and positions for signals of opportunity.

- Positions and identification of environmental features.
- Terrain height and calibration parameters.

Relative Positioning: Participants measure their positions relative to each other using proximity, ranging, or angular data. This is especially useful when there is insufficient information available to determine a stand-alone position solution.

2.2.4.6 Fault Detection Integrity monitoring ensures a reliable navigation solution by detecting, isolating, and excluding faults in hardware or software. In safety-critical applications, such as aviation, formal integrity systems are essential to meet performance standards.

2.3 GPS segments

The GPS system consists of *three segments*:

1. **Space segment** (satellites with atomic clocks)
2. **Control segment** (ground stations for clock offsets)
3. **User segment** (receivers)

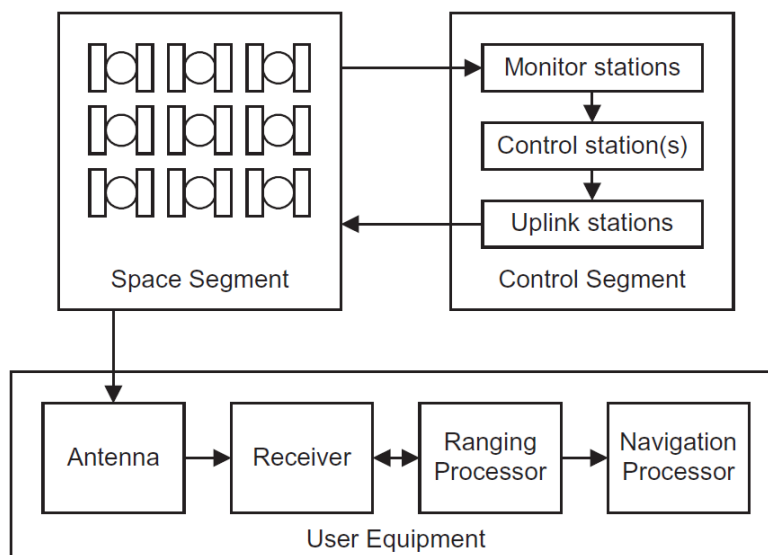


Figure 6: GPS segments

GNSS orbital planes are **inclined** with respect to the equator (at 55° for GPS). All the satellites form a **constellation**.

2.4 Radio Signal

2.4.1 GPS L1 CA-signal

The GPS radio signal is modulated through **Biphase shift key (BPSK) modulation**, with the amplitude of the signal given by:

$$s(t) = \sqrt{2P}C(t)D(t) \cos(2\pi f_{ca}t + \phi_0)$$

with:

- P : the signal power.
- $C(t)$: the **spreading code** (± 1). It is also called the **Pseudo Random Noise (PRN)** and is unique to each satellite, publicly available.
- $D(t)$: the **navigation data** (± 1). It contains the satellite orbit and clock information.
- f_{ca} : the **carrier frequency**. It is in the L-band between 1 and 2 GHz.
- t : time.
- ϕ_0 : phase offset.

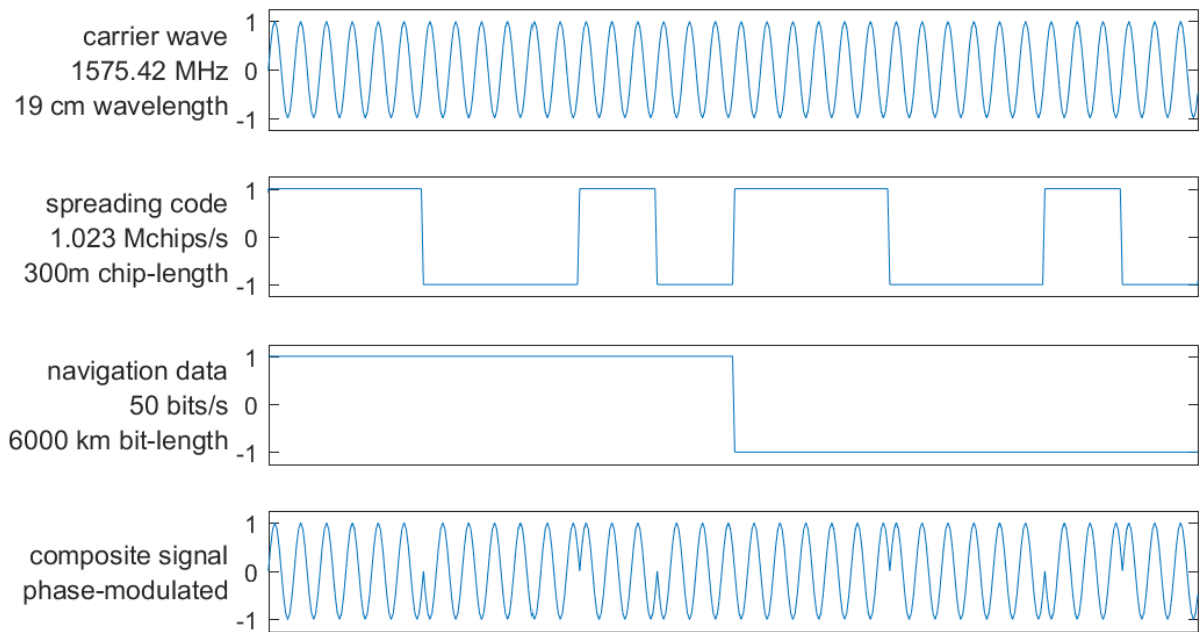


Figure 7: GPS L1 CA-signal (scale is not accurate)

2.4.2 Message Format

There are two main formats of **navigation data message**:

- **Fixed frame**: data always transmitted in same order with same repetition intervals
- **Variable frame**: a serie of fixed-length messages may be transmitted in any order

2.5 Initialisation

When starting, GPS receivers try to find a particular GPS satellite on *each of their channels* (tens to hundreds). This is done by **overlaying the received signal** with a replica of the **spreading code** and then shifting it until correlation shows a maximum (best fit, or match).

2.6 Pseudorange Measurement

The **pseudorange** $p_{r,s}$ is calculated by multiplying the travel time $\tau_{r,s}$ by the speed of light c :

$$p_{r,s} = c \cdot \tau_{r,s} \text{ where } \tau_{r,s} = t_r - t_s$$

with:

- t_s : signal transmission time (from satellite s)
- t_r : time of signal arrival (determined by receiver clock)

There are then two situations:

- **Signal acquisition**: pseudo-range prediction unknown, receiver-generated spreading code searched until correlation peak is found
- **Signal tracking**: pseudo-range prediction known, only vary the receiver-generated code phase slightly

Perceived carrier frequency varies due to: **Doppler effect** and **receiver clock drift**.

A **GNSS navigation solution** is $4D$ with three position dimensions and one time dimension. For any satellite, the pseudo-range measurement, corrected for satellite clock error (and other known errors):

$$\rho(t_{s,a}) = \sqrt{(r_s(t_{s,t}) - r_a(t_{s,a}))^T \cdot (r_s(t_{s,t}) - r_a(t_{s,a}))} + \delta\rho(t_{s,a})$$

with:

- $r_s(t_{s,t})$: satellite position at time of signal transmission
- $r_a(t_{s,a})$: user antenna position at time of signal arrival
- $\delta\rho(t_{s,a})$: receiver clock offset

2.7 Carrier Phase Measurement

Carrier Phase Measurement:

- Measures **fractional phase difference** between the received *carrier wave* from the satellite and a locally generated *replica*.
- Provides a **very precise distance** measure (satellite to receiver)
- Needs to be **initialized** by finding the initial number of carrier wave cycles.
- Is much more precise than pseudorange code measurement. thanks to the **carrier period** being **much smaller** than code chip duration (in L1 CA-code signal, *1540 carrier periods* fit in one PRN spreading code chip).

2.8 Jamming and Spoofing

There are multiple ways a GNSS signal may be threatened, jamming and spoofing being intentional attacks.

2.8.1 Jamming

By the time GNSS signals arrive at the antennas of a GNSS positioning system, the power level of these signals is very low. This low power level makes the signals susceptible to interference from other signals transmitted in the GNSS frequency range.

Jamming is a special case of signal interference where an attacker tries to block the incoming GNSS signal to a specific person/area.

GNSS receivers can use several methods to protect against interference and jamming:

- Signal filtering
- Multiple navigation sensors. For short-term interference, other sensors can help the receiver bridge brief periods of GNSS outage.
- Multi-frequency/multi-constellation GNSS makes it much harder to jam a signal on multiple different frequencies at once.
- Anti-jam antennas use multiple antenna elements to control the amount of signal received from a particular direction. When an anti-jam system senses interference from a direction, it turns down the antenna gain for it.

2.8.2 Spoofing

Unlike interference where GNSS is denied by overpowering the satellite signal, spoofing tricks the receiver into reporting an incorrect position. Spoofing is done by first jamming the GNSS receiver and then providing a false satellite signal that is either created by a signal generator or is a rebroadcast of a pre-recorded GNSS signal. Unlike interference, spoofing is always an intentional attack.

To protect against spoofing the same methods apply as against interference. Additionally, one of the most effective ways to protect against spoofing is to track encrypted signals that are broadcast by several of the GNSS constellations. Access to the encrypted signals is restricted and not available to all users.

2.8.3 Signal blockage

The GNSS signal can be blocked by many objects like trees or buildings, especially in urban areas. The main protection is again using multiple constellations and using additional sensors like an IMU.

2.8.4 Constellation failure

Although it is extremely unlikely that an entire constellation will fail, receivers that can track more than one constellation protect against this unlikely scenario.

3 GNSS performance

3.1 Error Sources

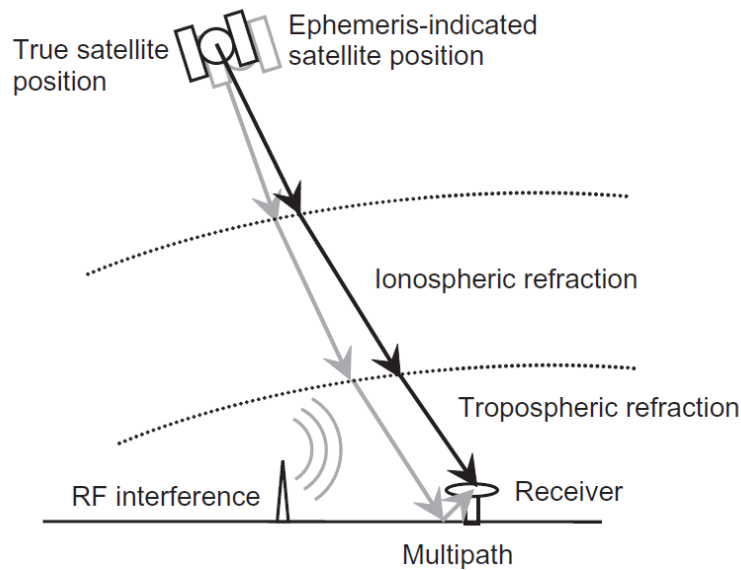


Figure 8: Principle sources of GNSS errors

3.1.1 Pseudorange Calculation

Multiple issues affect the calculation of the pseudorange:

- **satellite clock offset** (known).
- **receiver clock offset** (unknown).
- **ionosphere delay** (unknown).
- other errors, such as *multipath* (unknown).

The calculation is very sensible since $c \approx 3 \times 10^8$ m/s, and a **1 μ s** error will cause a **300 m** error in the calculated distance, since we have:

$$p_{r,s} = r_{r,s} + c \cdot (\delta t_s - \delta t_r)$$

where:

- $p_{r,s}$: pseudorange
- $r_{r,s}$: actual range
- δt_s : satellite clock offset
- δt_r : receiver clock offset

3.1.2 Ionosphere Delay

Ionospheric delay:

- Is due to **free electrons** in the ionosphere.
- Is highly variable (depends on **time** and **space**).
- Ranges from *a few meters to hundreds of meters*.
- Is maximum near geomagnetic equator, around local noon and during solar maxima.
- Is proportional to $1/\text{frequency}^2$.
- Can be estimated using two frequencies. This is why satellites emit at **L1** (1575.42 MHz) and **L2** (1227.60 MHz).

3.1.3 Masking Angle

GNSS receivers **ignore signals** from below a certain elevation, making them prone to errors (typically between 5° and 15°).

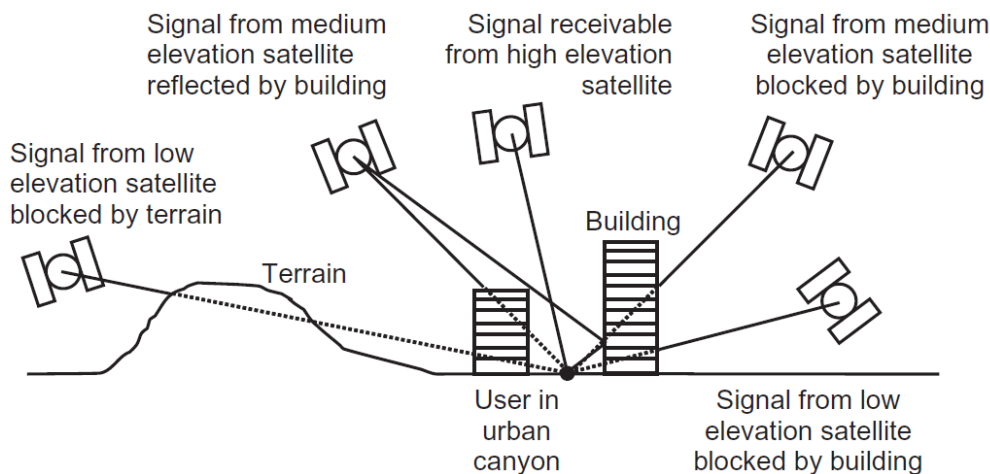


Figure 9: Errors due to terrain, buildings, and elevation angle

3.1.4 GNSS Augmentation Systems

GNSS augmentation systems supply differential corrections and integrity alerts that meet the needs of safety-critical applications. There are two types:

Criteria	Space-based augmentation systems (SBAS)	Ground-based augmentation systems (GBAS)
<i>Coverage</i>	Large country or small continent	Local area (e.g. an airfield)
<i>Broadcast</i>	Geostationary satellites	Ground-based transmitters
<i>Precision</i>	Lower than GBAS	Higher than SBAS

3.2 Accuracy and Precision

Accurate results may not be precise and precise results may not be accurate

- Accuracy = how close is the measurement to the actual value, can only be calculated if the ground truth is known
- Precision = how close is the measurement to other measurements, might be high even though some systematic error causes all measurements to be off

The quality of the measurement can be assessed through the carrier-to-noise-density ratio C/N_0 (signal strength).

The precision of the measurement depends on the method used:

Table 2: Precision of GNSS measurements

	Pseudorange	Carrier Phase
Precision	Few meters to few decimeters	Few centimeters to millimeter

3.3 Dilution of Precision

The geometry of the visible satellites affects accuracy. Poor satellite geometry (e.g. satellites clustered together) increases DOP, leading to less precise positioning. Good geometry (satellites spread out) reduces DOP and improves accuracy.

$$\text{Position Error} = \text{DOP} * \text{Range Error (UERE)}$$

If DOP is very high, the inaccuracy of the computed position will be much larger than the inaccuracy of the range measurement.

DOP Formula:

$$\sigma_G = \sqrt{\sigma_E^2 + \sigma_N^2 + \sigma_U^2 + \sigma_T^2}$$

Where

- σ_G is the GDOP (Geometric Dilution of Precision) value
- $\sigma_E^2, \sigma_N^2, \sigma_U^2$ are the East, North and Up variance components of receiver position estimate
- σ_T^2 is Variance of receiver clock offset estimate

If DOP is very high, the inaccuracy of the computed position will be much larger than the inaccuracy of the range measurement

Types of DOP:

- GDOP (Geometric Dilution of Precision)
 - Amplifies pseudorange error of the position determination
 - $GDOP^2 = PDOP^2 + TDOP^2$
- PDOP (Position Dilution of Precision) - 3D uncertainty representation
 - $PDOP^2 = HDOP^2 + VDOP^2$
 - $\sigma_P = \sqrt{\sigma_E^2 + \sigma_N^2 + \sigma_U^2}$
- HDOP (Horizontal Dilution of Precision) - 2D uncertainty representation
 - $\sigma_H = \sqrt{\sigma_E^2 + \sigma_N^2}$
- VDOP (Vertical Dilution of Precision)
 - Typically, $VDOP > HDOP$ because satellites ‘below’ the receiver cannot be acquired
 - This disparity increases with latitude due to less available satellites high in the sky
 - * GPS constellation’s orbit inclination is 55°
 - * There will never be a GPS satellite directly overhead at latitudes $> 55^\circ$ N/S
- TDOP (Time Dilution of Precision)

Assume only 4 satellites are being observed by a receiver. Maximising the tetrahedron formed by the 4 satellite points tends to minimize GDOP.

Conversely, DOP can be infinitely large (eg: 999.0 in assignment) when receiver-satellite vectors lie in the same plane. When this happens, the solution cannot distinguish between error in receiver clock and error in receiver position.

DOP values represent accuracy, with high DOP leading to poor precision. Impact of satellite position. DOP values change with time of day and location. For example, in high-latitude areas or urban environments, poor satellite geometry can lead to high DOP and lower positioning accuracy.

3.4 Availability, Continuity and Integrity

- Availability: the percentage of time that a sufficient amount of satellites have unblocked direct lines of sight (LOSs).
- Continuity: the ability of the total navigation system to continue to perform its function during the intended operation. Continuity is critical whenever reliance on a particular system is high. For a pilot during an instrument approach procedure, continuity and integrity are vital.
- Integrity: how much the information supplied by the system can be trusted to be correct. This requires the system to provide timely warnings to the user when the equipment is unreliable for navigation purposes—due to obstructions, jamming, multipath, or any other event that degrades accuracy. Almanac: contains information about which satellite is where at which time

3.5 PPP & RTK

3.5.1 Abbreviations

- **SV**: space vehicles or orbiting space vehicles
- **RTK**: Real-Time Kinematic
- **PPP**: Precise Point Positioning
- **PPP-RTK**: Hybrid of PPP and RTK
- **CORS**: Continuously Operating Reference Station
- **NRTK**: Network RTK
- **OSR**: Observation State Representation
- **SSR**: State Space Representation

3.5.2 PPP

- **PPP** achieves decimetre-level or better accuracy by leveraging corrections transmitted via satellite or the internet.
- It utilises the **SSR** message format for efficient data transmission.
- **PPP** is suitable for global applications due to its independence from regional base stations.
- The primary limitation of **PPP** is its long convergence time, typically ranging from 5 to 30 minutes.
- **PPP** primarily corrects for orbit errors, clock errors, and biases to achieve its positioning solution.
- **PPP** offers a trade-off between accuracy and coverage, providing moderate accuracy over a wide area.
- Variations like PPP-AR and A-PPP exist, offering enhanced accuracy or specialized capabilities.

3.5.3 RTK

- **RTK** provides centimetre-level accuracy, achieving the highest precision among the discussed technologies.
- **RTK** relies on the **OSR** message format, which requires a two-way communication channel between the base station and the rover.
- The coverage area of **RTK** is limited to a short range (30-50 km) due to signal degradation with distance.
- **RTK** boasts a near-instantaneous convergence time, typically under 5 seconds.
- **RTK** corrects for various errors, including orbit errors, clock errors, bias, ionospheric delay, and tropospheric delay.
- **RTK** is widely adopted in applications demanding high accuracy within a limited area, such as surveying and agriculture.
- Developments like Network RTK (NRTK) address range limitations by incorporating networks of base stations.

3.5.4 PPP-RTK

- **PPP-RTK** combines the strengths of PPP and RTK, offering high accuracy, global coverage, and fast convergence.
- **PPP-RTK** achieves centimetre-level accuracy comparable to RTK while offering global coverage.
- **PPP-RTK** employs the efficient **SSR** message format, enabling broadcast corrections and lower bandwidth requirements.
- **PPP-RTK** utilises a network of CORS stations for precise atmospheric and clock corrections.
- **PPP-RTK** converges significantly faster than PPP, typically within 1-10 minutes, and potentially seconds under ideal conditions.
- It effectively corrects for orbit errors, clock errors, bias, ionospheric delay, and tropospheric delay, allowing for integer ambiguity resolution.
- **PPP-RTK** gracefully degrades to standard PPP performance when outside the range of the CORS network.

3.5.5 Comparing RTK, PPP, and PPP-RTK

Feature	RTK	PPP	PPP-RTK
Accuracy	cm-level (up to 1 cm + 1 ppm)	dm-level or better (less than 10 cm)	cm-level , similar to RTK
Coverage Area	Limited range (typically 30-50 km from the base station)	Global	Global with graceful degradation to standard PPP outside the range of the CORS network
Message Format	OSR (Observation Space Representation)	SSR (State Space Representation)	SSR (State Space Representation)

Feature	RTK	PPP	PPP-RTK
Transmission Channel	Two-way communication between base station and rover	Corrections delivered via satellite or the internet	Corrections broadcast to users , enabling a large number of users to connect simultaneously
Convergence Time	Near-instantaneous (typically less than 5 seconds)	Relatively long (typically 5-30 minutes)	Fast (typically 1-10 minutes, potentially within seconds under ideal conditions)
Errors Solved	Orbit errors, clock errors, bias, ionospheric delay , tropospheric delay	Orbit errors, clock errors, bias	Orbit errors, clock errors, bias, ionospheric delay , tropospheric delay , enabling integer ambiguity resolution
Key Strengths	High accuracy, very fast convergence time	Global coverage, no reliance on local base stations	High accuracy, fast convergence time, global coverage, lower bandwidth requirements compared to RTK, graceful degradation outside CORS range
Key Limitations	Limited range, high bandwidth requirements, reliance on local base stations	Long convergence time, lower accuracy compared to RTK	Still requires a CORS network (though less dense than RTK) and may degrade to standard PPP with increasing distance from CORS station

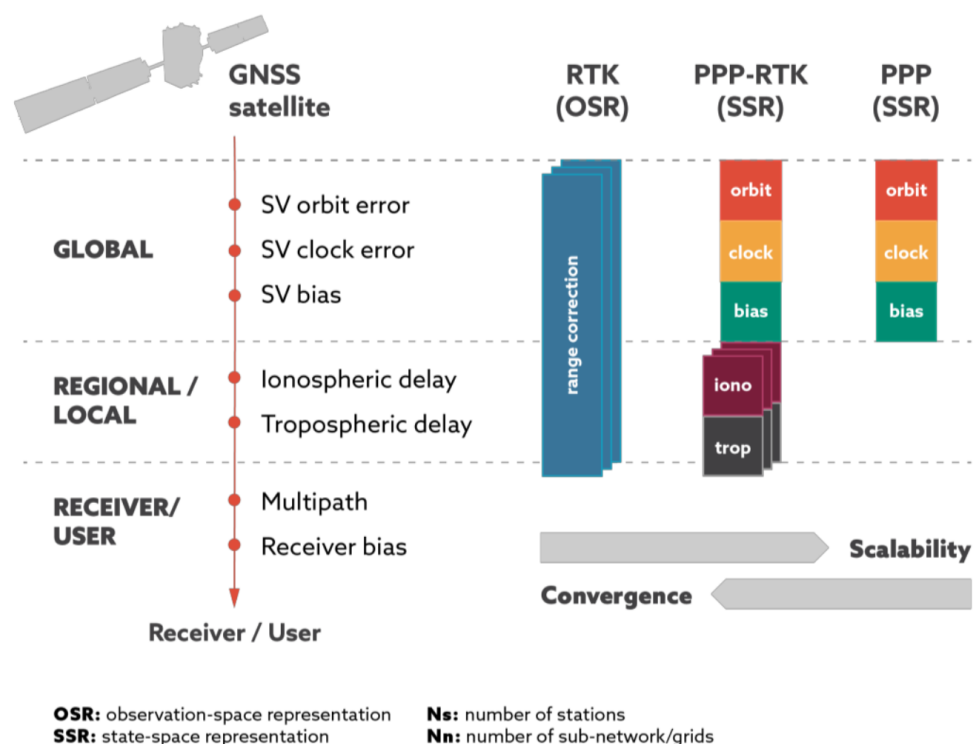


Figure 10: Difference in message format and resolved errors

3.6 Differential GNSS (DGNSS)

DGNSS improves navigation accuracy by calibrating **correlated range errors** using measurements from a **reference station** at a pre-surveyed location. However, **tracking errors**, **multipath effects**, and **NLOS reception errors** remain uncorrected since they are uncorrelated between locations.

3.6.1 How DGNSS Works

- A **reference station** transmits correction data via a data link to a **nearby rover** (within 5–10 km).
- **Key steps:**

1. **Baseline computation:** Differences in observations between the reference and rover are used to calculate the vector between them.
2. **Rover position:** Adding the baseline vector to the reference station's known coordinates gives the rover's position.

3.6.2 Local and Regional DGNSS

3.6.2.1 Local Area DGNSS (LADGNSS)

- Corrections from a **single reference station** are transmitted to users (rovers) within the station's range.
- User computes relative clock offsets and corrects errors for the selected satellites.

3.6.2.2 Regional Area DGNSS (RADGNSS)

- Enhances LADGNSS by using corrections from **multiple reference stations**.
- Increases accuracy for wider regional coverage.

3.6.3 Wide Area DGNSS (WADGNSS)

- Provides **meter-level accuracy** over large areas (e.g., continents or countries).
- Uses **satellite broadcasts** for corrections instead of terrestrial or network-based transmissions.
- **Corrections process:**
 - **Reference stations** send pseudo-range and ionosphere delay measurements to a Master Control Station (MCS).
 - The MCS computes corrections for satellite ephemeris, clock parameters, and ionosphere data, then broadcasts them via satellites.

3.6.4 Relative GNSS (RGNSS)

- Focuses on accurate positioning **relative to a reference station** rather than absolute accuracy relative to Earth.
- Computes the **baseline vector** by differencing pseudo-range measurements from the reference and rover.

4 CRS

4.1 Coordinate Systems

4.1.1 Coordinate Reference Systems

According to the EPSG and ISO guidelines, four groups of CRSs can be distinguished:

- **Geographic CRSs** give geographic coordinates in degrees (2D latitude and longitude, and for 3D also height) relative to an ellipsoidal model of the Earth's surface. Examples include the European ETRS89, the global ITRF2020, and WGS 84. Geographic CRSs, such as ETRS89 and WGS 84, cannot be visualized on a flat plane without a map projection.
- **Projected CRSs** present geo-information on a flat surface in Cartesian (x and y) coordinates. A projected CRS is a derivative of a geographic CRS, where a map projection is used for the depiction on the flat surface. An example of this is the Dutch RD with the geographic CRS RD-Bessel as its basis.
- **Vertical CRSs** for recording height and depth relative to a reference plane. This reference plane is often based on the direction of gravity and normally does not coincide with the surface of an ellipsoid. With a reference plane based on the direction of gravity, no water flows between two points of equal height, but water usually does flow between two points of equal ellipsoidal height. Examples of vertical CRSs are the NAP and the LAT.
- **Compound CRSs** are composite CRSs, for example: RDNAP which is composed of the projected CRS RD and the vertical CRS NAP.

In this chapter of the summary, we will highlight two examples of Coordinate Reference Systems. Before doing so, here is an overview of some terminology and their examples:

Geodetic terminology:

- Terrestrial Reference System (definition) (e.g. ETRS89, WGS84)

- Terrestrial Reference Frame (realisation) (e.g. ETRF2000, ITRF2020)
- Ellipsoid (e.g. GRS80, Bessel 1841)
- 'Coordinate notation' (e.g. axis, order, units)
- Map projection (optional) (e.g. UTM, LCC)
- Epoch (for time-dependent coordinates) (e.g. 2024.91 (27 November 2024))

NB: be aware of the difference between epoch and the year in names!

Geo-spatial terminology:

- Datum ensemble != Terrestrial Reference System
- Datum = Terrestrial Reference Frame + ellipsoid
- Coordinate system = Coordinate notation (+ map projection)
- CRS (= projection in some software) = datum + coordinate system

4.1.2 Geographic Coordinate Reference Systems

The use of Geographic Coordinate Reference Systems is very common. They use degrees of latitude and longitude and sometimes also a height value to describe a location on the earth's surface.

Lines of latitude run parallel to the equator and divide the earth into 180 equally spaced sections from North to South. The reference line for latitude is the equator and each **hemisphere** is divided into ninety sections, each representing one degree of latitude. Wherever you are on the earth's surface, the distance between the lines of latitude is the same (60 nautical miles).

Lines of longitude, on the other hand, do not stand up so well to the standard of uniformity. Lines of longitude run perpendicular to the equator and converge at the poles. The reference line for longitude (the prime meridian) runs from the North Pole to the South Pole through Greenwich, England. At the equator, and only at the equator, the distance represented by one line of longitude is equal to the distance represented by one degree of latitude.

Using the geographic coordinate system, we have a grid of lines dividing the earth into squares that cover approximately 12363.365 square kilometres at the equator — a good start, but not very useful for determining the location of anything within that square. To be truly useful, a map grid must be divided into small enough sections so that they can be used to describe (with an acceptable level of accuracy) the location of a point on the map. To accomplish this, degrees are divided into **minutes** (') and **seconds** ("). There are sixty minutes in a degree, and sixty seconds in a minute (3600 seconds in a degree). So, at the equator, one second of latitude or longitude = 30.87624 meters.

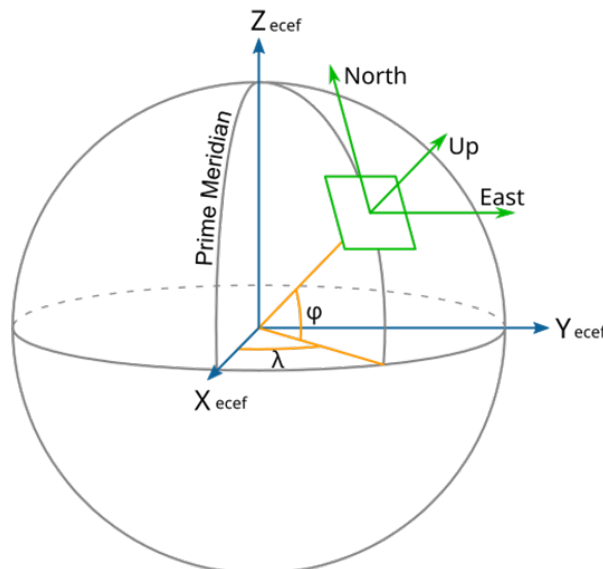


Figure 11: Fundamentals of a Geographic coordinate system

4.1.3 Projected Coordinate Reference Systems

There is a difference between a geographic coordinate system (GCS) and a projected coordinate system (PCS). In short, a GCS defines **where** the data is located on the earth's surface; a PCS tells the data **how** to draw on a flat surface. A

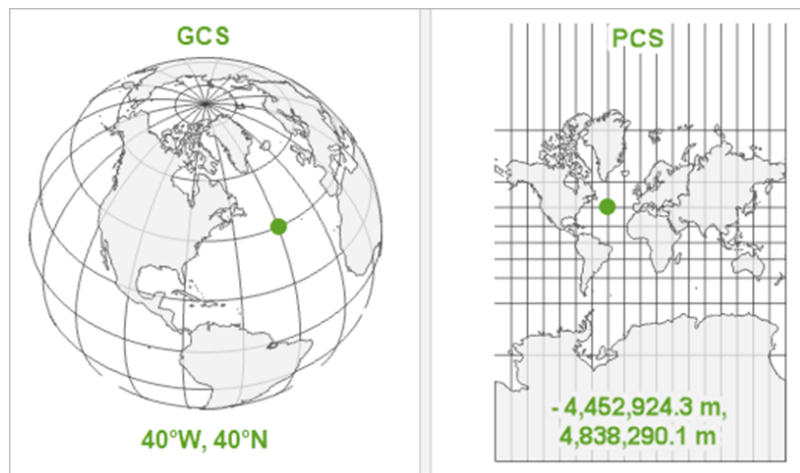


Figure 12: Geographic coordinate systems vs Projected coordinate system

GCS is round, and so records locations in angular units (usually degrees). A PCS is flat, so it records locations in linear units (usually meters).

The GCS is what ties your coordinate values to real locations on the earth. Only knowing the latitude and longitude of a location is thus not good enough, as it only tells you where a location is within a GCS. To draw a graticule, you need a model of the earth that is at least a regular spheroid, if not a perfect sphere. There are many different models of the earth's surface, and therefore many different GCS!

Once you know where to draw something, you need to know how. The earth's surface — and your GCS — are round, but the map is flat. This is where the map projections come in. They tell you how to distort the earth so the parts that are most important to your map get the least distorted and are displayed best on the flat surface of the map. A **projected coordinate system** (PCS) is a GCS that has been flattened using a map projection.

Your data must have a GCS before it knows where it is on Earth. Projecting your data is optional, but projecting your map is not. Maps are flat, so your map must have a PCS to know how to draw.

Coordinates in a PCS are recorded in a **Linear Unit**, often meters. A PCS also contains a Geographic Coordinate System (e.g. WGS 1984)! Remember that a PCS is just a GCS that has been projected.

Coordinates in a GCS are recorded in an Angular Unit, usually degrees. The **Prime Meridian** is an arbitrary line of longitude that is defined as 0° . The **Datum** defines which model is used to represent the earth's surface and where that model is positioned relative to the surface. The **Spheroid** is the regular model of the irregular earth. It's part of the datum. **Semimajor Axis**, **Semiminor Axis**, and **Inverse Flattening** define the size of the spheroid.

4.1.4 Linear Reference Systems

Linear referencing is the method to store and geographically locate data using relative positions along a measured line feature without the need to explicitly use x,y coordinates or an address. When data is linearly referenced, measure values are used to measure the distance along a line feature, allowing multiple sets of dynamically changing attribute data to be associated with any portion of an existing linear feature, independent of its beginning and end. Linear referencing is used for many reasons. The following are the two primary reasons:

- Many locations are recorded as events along linear features.

For example, locations of traffic accidents are recorded using a convention such as “27 meters east of reference mile marker 35 along State Highway 287.” Many sensors record conditions using measures of distance or time along the lines—along pipelines, along roads, along streams, and so forth.

- Linear referencing is also used to associate multiple sets of attributes to portions of linear features without requiring that underlying lines be segmented (split) each time that attribute values change.

For example, most road centreline feature classes are segmented where three or more road segments intersect and where the road names change.



Figure 13: Hectometre marker

Great examples of Linear Reference Systems are the Dutch **hectometre markers**. As the name suggests, Dutch hectometre markers are spaced at 100-metre intervals. In addition to showing the motorway number and location, they also bear a **carriageway identifier** – Li for Links (Left) and Re for Rechts (Right). The carriageways are identified as being left-hand and right-hand as viewed by somebody looking in the direction of increasing location numbers. By and large, Dutch location numbers increase as one moves away from Amsterdam, or in the case of roads that do not originate in Amsterdam, location numbers increase as one moves eastwards away from the North Sea.

Dynamic segmentation is the process of computing the map locations of events stored and managed in an event table and displaying them on a map using route features. The term dynamic segmentation is derived from the concept that line features need not be split (in other words, segmented) each time an attribute value changes; you can dynamically locate the segment.

Using dynamic segmentation, multiple sets of attributes can be associated with any portion of an existing linear feature independently of where it begins or ends. These attributes can be displayed, queried, edited, and analyzed without affecting the underlying linear feature's geometry.

4.2 Terrestrial Reference Systems and Frames

In this chapter of the summary, we will highlight some examples of Coordinate Reference Systems and Frames:

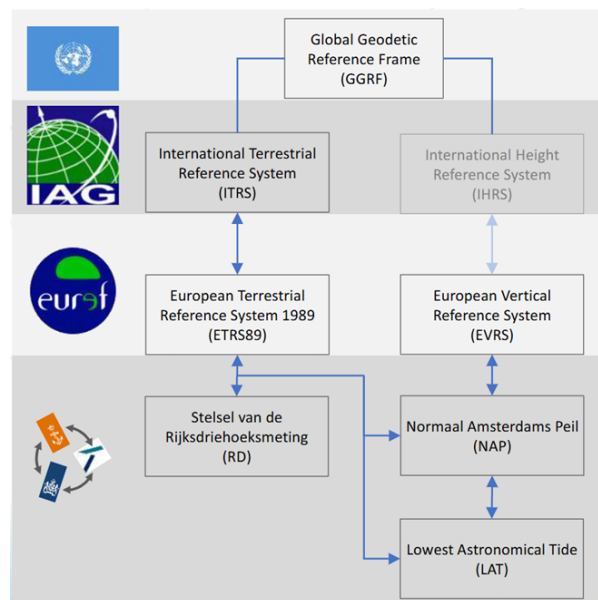


Figure 14: Terrestrial Reference Systems

ITRS: International Terrestrial Reference System uses the GRS80 ellipsoid, with as its origin the centre of mass of the Earth. Greenwich (UK) is used as a meridian. When using time-dependent coordinates, you always need to specify the epoch (e.g. @2024.91).

ITRF: International Terrestrial Reference Frames are realisations of the ITRS. They publish updates every ~1-6 years. Due to the movement of tectonic plates, the differences between each iteration can be multiple centimetres to decimetres. Two examples are ITRF2014 and ITRF2020 (NB: Frame != epoch, e.g. ITRF2014@2022.90). These updates reflect:

- Improved precision of the station positions $r(t_0)$ and velocities $\dot{r}$ due to the availability of a longer period of observations, which is particularly important for the velocities,
- Improved datum definition due to the availability of more observations and better models,
- Discontinuities in the time series due to earthquakes and other geophysical events,
- Newly added and discontinued stations,
- Occasionally a new reference epoch t_0 .

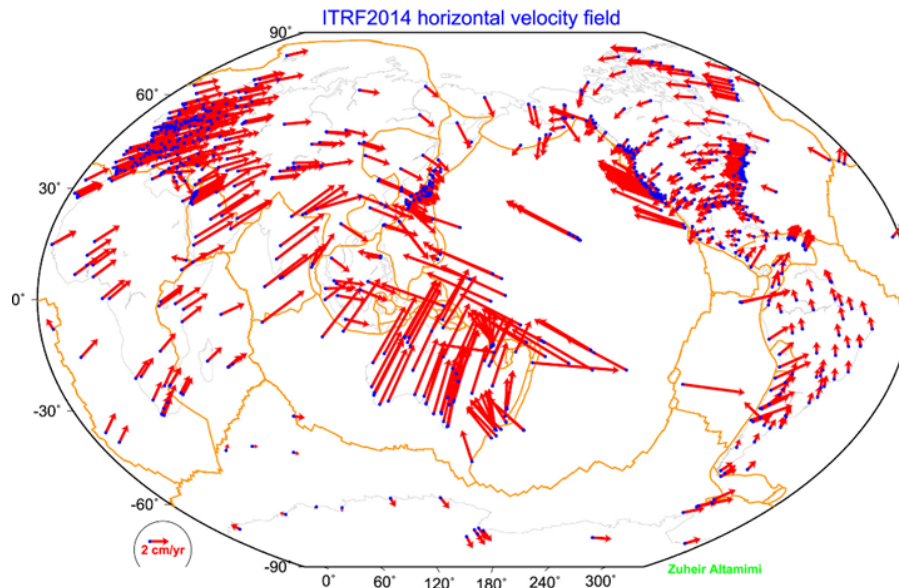


Figure 15: Changes in ITRF2014

WGS 84 is aligned with ITRS (WGS 84-G2296 (2024) = ITRF2020). For time-dependent coordinates, however, it has limited precision. The realisations (frames) often have differences between m – cm. The ensemble code (+/- 2m): EPSG:4326 for 2D (often used as unknown latlon).

ETRS89: European Terrestrial Reference System 1989 is ITRF89 at epoch 1989.00. It moves with a stable part of Europe, “no” time-dependency. A new realisation (frame) comes with every ITRF. ETRF2000 is recommended for mapping and the ensemble code has an accuracy of +/- 0.1m.

A transformation from ETRS89 to ITRS (and WGS84) is an example of a **Time-dependent transformation** (2.4 cm/year), but a specification of realisation and epoch is needed and most software does not include this transformation. A **Null transformation** is possible, simple and the current practice, but it is not future-proof, since the difference is growing (now 0.9m).

4.3 Datum and Transformations

4.3.1 Transformations and conversions

The International Association of Oil and Gas Producers (EPSG) used a *de facto* standard instead of an ISO standard. The EPSG collects all the different reference systems and their transformations. There are two different steps when working with 3D data. First, the coordinates need to be converted to a new coordinate system, after which the height values are transformed.

Note, that a geoid is a stochastic height transformation and not a conversion. It uses in total 14 parameters:

- 7 parameters;
 - 3 Translations [m];
 - 3 Rotations [“];
 - 1 Scale factor [ppm];
- 7 rates and reference epoch (t_0).

These parameters can be given (by an official source), user-estimated (empirical) and/or conventional (by definition). A correction grid can be used as an alternative transformation or an additional conversion.

A big difference between datum transformations and coordinate conversions is that the parameters for the datum transformation are often empirically determined and thus subject to measurement errors, whereas coordinate conversions are fully deterministic. More specific, three possibilities need to be distinguished for the datum transformation parameters:

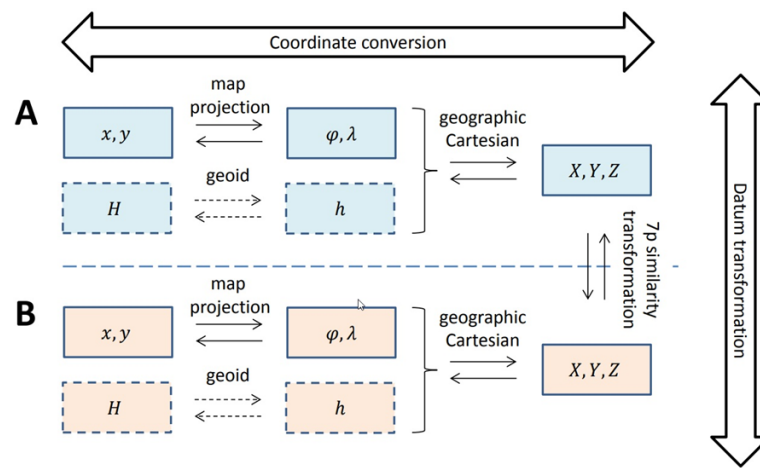


Figure 16: Map of coordinate conversion and datum transformation

1. **The datum transformation parameters are conventional.** This means they are chosen and therefore not stochastic. The datum transformation is then just some sort of coordinate conversion (which is also not stochastic).
2. **The datum transformation parameters are given but have been derived by a third party through measurements.** This third party often does new measurements and updates the transformation parameters occasionally or at regular intervals. This is also related to the concepts of reference systems and reference frames. Reference frames are considered (different) realisations of the same reference system, with different numerical values assigned to the coordinates of the points in the reference frame, and often with different realisations of the transformation parameters. The station coordinates and transformation parameters are stochastic, so new measurements, mean new estimates that are different from previous estimates.
3. **There is no third party that has determined the transformation parameters, and you as a user, have to estimate them using at least three common points in both systems.** In this case you will need coordinates from the other reference system. Keep in mind that the coordinates from the external reference system should all come from the same realization, or, reference frame.

4.4 Datums

When people in the field of Geomatics are talking about height, they can reference multiple different definitions of height:

- **Geometric:** Ellipsoidal height (max. MSL deviation $\pm 150\text{m}$)
- **Physical:** Height above the geoid $\approx$ Mean Sea Level (MSL)
- **Relative:** Height above ground level (DTM)
- **Water depth:** Lowest Astronomical Tide (LAT)

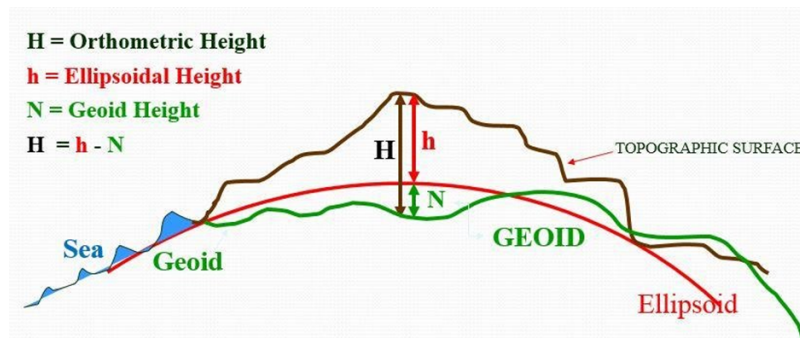


Figure 17: Physical heights in Geomatics

There are multiple physical height standards depending on where you want to know the height:

- **International Height Reference Systems (IHRs)** – No realisation yet
- **European Vertical Reference System (EVRS)** – Realisations available, but not widely used yet (dm – cm)
- **Earth Gravitation Model (EGM84, 96, 2008, 2020)** – By USA like WGS84 (m – dm)
- **National Height Systems** – Based on local MSL

4.5 Map Projections

Map projections try to portray the surface of the earth or a portion of the earth, on a flat piece of paper or computer screen. They try to transform the earth from its spherical shape (3D) to a planar shape (2D).

A **coordinate reference system** (CRS) then defines how the two-dimensional, projected map relates to real places on Earth. The decision of which map projection and CRS to use depends on the regional extent of the area you want to work in, on the analysis you want to do, and often on the availability of data.

When viewed at close range the earth appears to be relatively flat. Maps are representations of reality: they are designed to not only represent features but also their shape and spatial arrangement. Each map projection has **advantages** and **disadvantages**.

The best projection for a map depends on the **scale** of the map, and on the purposes for which it will be used. For example, a projection may have unacceptable distortions if used to map the entire African continent but may be an excellent choice for a **large-scale (detailed) map** of your country.

Different projection methods can be produced by surrounding the globe in a **cylindrical** fashion, as a **cone**, or even as a **flat surface**. Each of these methods produces what is called a **map projection family**. Therefore, there is a family of **planar/azimuthal projections** (c), a family of **cylindrical projections** (a), and another called **conical projections** (b).

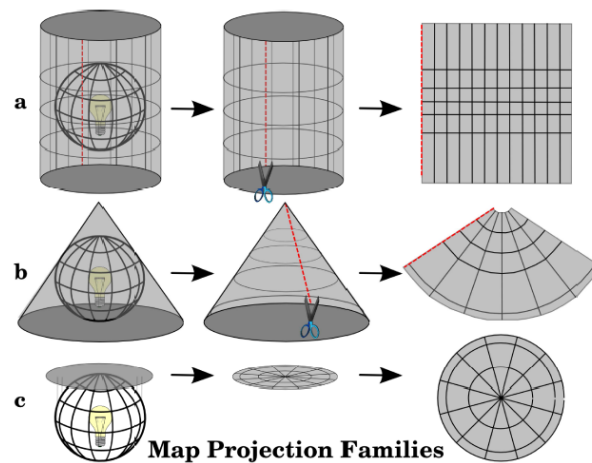


Figure 18: Map Projection Families

Map projections are never absolutely accurate representations of the spherical Earth. As a result of the map projection process, every map shows **distortions of angular conformity, distance and/or area**. It is usually impossible to preserve all characteristics at the same time in a map projection. This means that when you want to carry out accurate analytical operations, you need to use a map projection that provides the best characteristics for your analyses.

When working with a globe, the main directions of the compass (North, East, South and West) will always occur at 90 degrees to one another. In other words, the East will always occur at a 90-degree angle to the North. Maintaining correct **angular properties** can be preserved on a map projection as well. A map projection that retains this property of angular conformity is called a **conformal** or **orthomorphic projection**. These projections are used when the **preservation of angular relationships** is important. They are commonly used for navigational or meteorological tasks.

If your goal in projecting a map is to accurately measure distances, you should select a projection that is designed to preserve distances well. Such projections, called **equidistant projections**, require that the **scale** of the map is **kept constant**. A map is equidistant when it correctly represents distances from the centre of the projection to any other place on the map. **Equidistant projections** maintain accurate distances from the centre of the projection or along given lines. These projections are used for radio and seismic mapping, and navigation.

When a map portrays areas over the entire map, so that all mapped areas have the same proportional relationship to the areas on the Earth that they represent, the map is an **equal area map**. As the name implies, these maps are best used when calculations of area are the dominant calculations you will perform. If, for example, you are trying to analyse a particular area in your town to find out whether it is large enough for a new shopping mall, **equal area projections** are the best choice. On the one hand, the larger the area you are analysing, the more precise your area measures will be, if you use an equal area projection rather than another type. On the other hand, an equal area projection results in **distortions of angular conformity** when dealing with large areas.

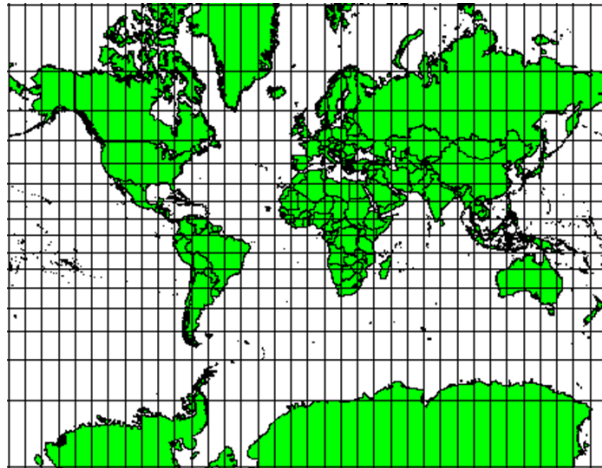


Figure 19: Conformal map projection

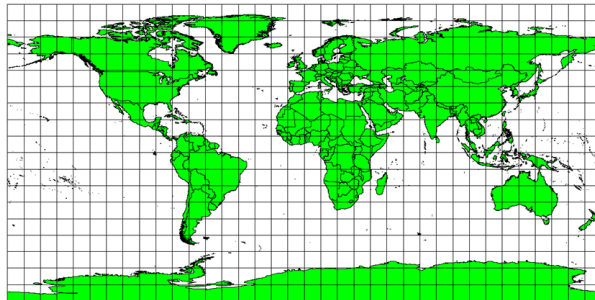


Figure 20: Equidistant map projection

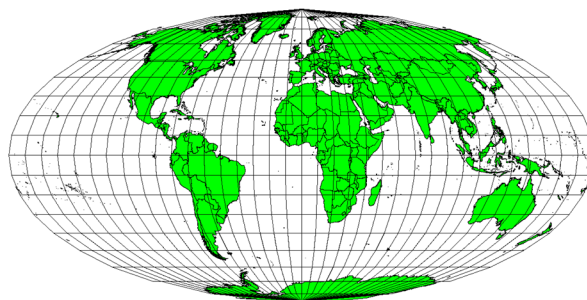


Figure 21: Equal area map projection

4.6 RDNAP

4.6.1 Coordinate Systems

Official 3D coordinate system of the Netherlands and Europe: European Terrestrial Reference System 1989 (ETRS89). ETRS89 is linked to the International Terrestrial Reference System (ITRS) by a time-dependant coordinate transformation. National coordinate systems in Europe are linked to ETRS89.

4.6.1.1 Rijksdriehoeksmeting (RD) Coordinates in the Dutch Stelsel van de Rijksdriehoeksmeting (RD) are the most-frequently used 2D coordinates on land and internal waters. RD coordinates are defined by the official transformation from ETRS89 coordinates. Maintaining reference points for ETRS89 and the transformation to RD coordinates are legal responsibilities of Kadaster.

4.6.1.2 Normaal Amsterdams Peil (NAP) Heights relative to Normaal Amsterdams Peil (NAP) are the official and the most-frequently used heights on land and internal waters. The NAP is a legal responsibility of Rijkswaterstaat. Ellipsoidal heights in ETRS89 can be transformed with the quasi-geoid model to NAP with a precision higher than ETRS89 coordinates obtained with most GNSS measurements.

4.6.2 Coordinate transformation RDNAPTRANS™

The official coordinate transformation between European ETRS89 coordinates and Dutch coordinates in RD and NAP is called RDNAPTRANS™.

The recommended ETRS89 realisation is ETRF2000 at epoch 2010.50 (AGRS2010). When using RDNAPTRANS™2018 it is important to use this realisation and epoch, especially for the height. For applications demanding high accuracy, it is recommended to obtain the NAP height of the point of interest by levelling to nearby NAP benchmarks.

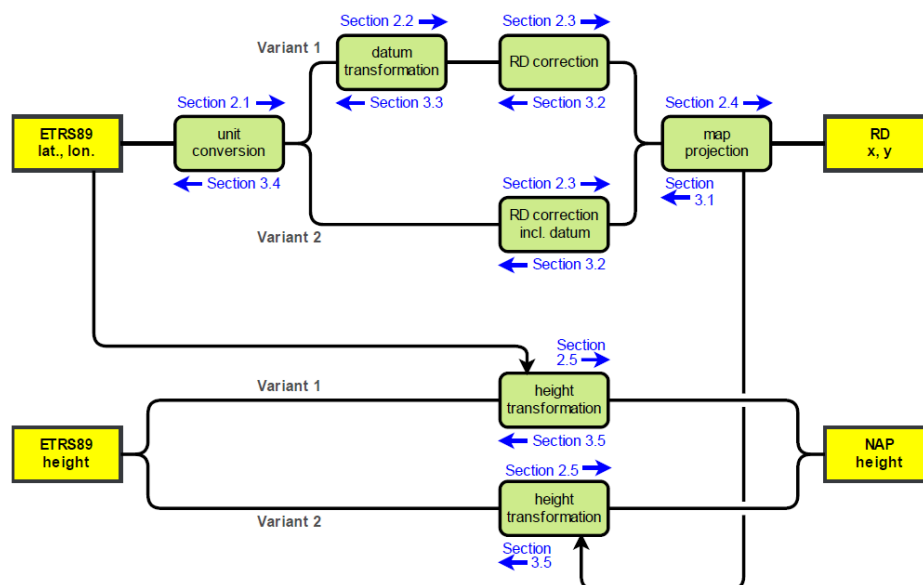


Figure 1.2.2. Guide to the sections (blue) of this document for the different steps of the transformation procedure.

Figure 22: Figure 1.2.2

There are two variants for the implementation of the horizontal component of RDNAPTRANS™2018 and two variants for the vertical component (Figure 1.2.2).

Implementation variant 1 applies the datum transformation as a separate step using a 3D similarity transformation. The advantage of implementation variant 1 is that it has no strict bounds for the area where horizontal coordinates can be transformed correctly. The disadvantage is that many software packages do not support implementation variant 1 for the horizontal component.

Implementation variant 2 includes the datum transformation in the correction grid and uses a different quasi-geoid grid for the height transformation. Implementation variant 2 for the horizontal component is supported by more software but can only be used within the bounds of the correction grid (Figure 1.1.1). The difference in the resulting coordinates between the two variants is well below 0.0010 m within the bounds of the RDNAPTRANS™2018 grids.

4.6.3 Transformation from ETRS89 to RD and NAP: Steps

1. Datum transformation
 - 1.1 Conversion to geocentric Cartesian coordinates. Variant 1, The ellipsoidal geographic ETRS89 coordinates of a point of interest must be converted to geocentric Cartesian ETRS89 coordinates to be able to apply a 3D similarity transformation. Variant 2, the datum transformation is included in the correction grid.
 - 1.2 3D similarity transformation
 - 1.3 Conversion from geocentric Cartesian coordinates back to ellipsoidal geographic Bessel coordinates (Formula 2.2.3).
2. RD correction
 - 2.1 Bilinear correction grid interpolation to obtain real Bessel coordinates.
 - 2.2 To transform the point of interest, Determine nearest grid points
 - 2.3 Iterative correction of the point of interest from pseudo Bessel coordinates to real Bessel coordinates,
 - 2.4 Datum transformation in the correction grid
3. Map projection
 - 3.1 Projection from ellipsoid to sphere (Gauss conformal projection from the ellipsoid to a sphere)
 - 3.2 Projection from sphere to plane
4. Height transformation
 - 4.1 Bilinear quasi-geoid grid interpolation
 - 4.2 Transformation to NAP

4.6.4 Transformation from RD and NAP to ETRS89: Steps

1. Inverse map projection
 - 1.1 Projection from plane to sphere
 - 1.2 Projection from sphere to ellipsoid
2. RD correction
 - 2.1 Direct correction
 - 2.2 Datum transformation in the correction grid
3. Datum transformation
 - 3.1 Variant 1, transformation from ellipsoidal geographic Bessel coordinates of a point of interest to ellipsoidal geographic ETRS89 coordinates. Variant 2 the datum transformation is included in the correction grid
 - 3.2 the ellipsoidal geographic Bessel coordinates of a point of interest must be converted to geocentric Cartesian Bessel coordinates
 - 3.3 The 3D similarity transformation must be applied to the geocentric Cartesian Bessel coordinates of the point of interest to obtain geocentric Cartesian ETRS89 coordinates.
 - 3.4 The geocentric Cartesian ETRS89 coordinates of the point of interest must be converted back to ellipsoidal geographic ETRS89 coordinates. The latitude is computed iteratively.
4. Conversion of radians or decimal degrees to decimal degrees
5. Height transformation: the physical NAP height of a point of interest to the purely geometrical ellipsoidal ETRS89 height, based on the quasi-geoid model NLGEO2018. The NAP height of the point of interest must be transformed to ellipsoidal ETRS89 height (Formula 3.5) using the interpolated quasi-geoid height of the point of interest.

5 Indoor Positioning

5.1 Spaces

Perceiving and describing space:

- Empty or containing things
- Unlimited or bounded
- Physical or imaginary

Cell: is a bounded portion of space (a space unit)

Space in Positioning and Localization: partitioning space from the **sensor reception perspective**

Space classification according to reception of GPS signal:

- **Open outdoors:** outside building, open sky condition, enough satellites for positioning
- **Semi-outdoors:** outside building, slight coverage (e.g. wooded area), some satellites availability
- **Light indoors:** inside building, slight coverage (e.g. areas around windows), some satellites availability
- **Deep indoors:** inside building, no satellite coverage

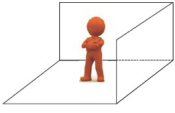
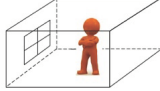
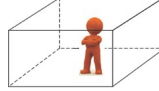
Environment	Open Outdoors	Semi-Outdoors	Light Indoors	Deep Indoors
Definition	Outside a building	Near a building	In a room with windows	In a room without windows
Example				

Figure 23: Types of spaces for GPS

Spaces are abstracted and represented using:

- Boundary Representation (BRep)
- Constructive Solid Geometry (CSG)
- Spatial Occupancy Enumeration

Navigation network (supported by Poincaré duality theory):

- **Nodes:** associated with space units, can contain semantic information about location
- **Edges:** represent connectivity between spaces
- **Costs** (of edges): indicate distance or travel time between nodes

Space partition in 3D:

- Bottom enclosure
- Side enclosure
- Top enclosure

Field	Classification	Physical Boundary
Navigation	Functional space, Object space, Remaining space & Indoor, Semi-indoor, Semi-outdoor, Outdoor	Architectural (e.g. wall, floor, roof, fence)
Positioning and Localization	Indoor, Semi-outdoor, Outdoor	Building, waterbody, bridge, tunnel

5.2 Wi-Fi-Based Approaches

Wi-Fi monitoring and fingerprinting are techniques used to gather information about wireless networks, but they differ in purpose, methodology, and application.

5.2.1 Wi-Fi Monitoring

- Main Idea: Wi-Fi monitoring involves passively observing and capturing Wi-Fi traffic (data packets) in the surrounding environment. This includes analyzing signals from access points (APs) and devices, such as SSIDs, signal strength, channel usage, and even packet contents (if not encrypted).
- Purpose: It's typically used for network troubleshooting, performance optimization, and security auditing.
- How It Works: A Wi-Fi adapter is set to monitor mode, allowing it to capture all wireless traffic in range, even if not destined for the monitoring device.
- Applications:
 - Analyzing traffic patterns and identifying potential interference.

5.2.2 Wi-Fi Fingerprinting

- Main Idea: Wi-Fi fingerprinting involves mapping and storing unique characteristics (or “fingerprints”) of Wi-Fi signals at different locations to determine a device's location or context later.
- Purpose: It's primarily used for location-based services and indoor positioning systems (IPS).
- How It Works:
 - Offline phase: Wi-Fi signals (like Received Signal Strength Indicator, or RSSI) are measured and recorded at various locations to create a “radio map.”
 - Online phase: The current Wi-Fi signal characteristics are compared to the radio map to estimate the device's location.
- Applications:
 - Indoor navigation and wayfinding (e.g., in malls, airports).
 - Asset tracking in warehouses.
 - Context-aware services like smart lighting or targeted advertisements.

5.2.3 Key Differences

Aspect	Wi-Fi Monitoring	Wi-Fi Fingerprinting
Objective	Traffic analysis, security, and troubleshooting	Location determination
Methodology	Passive traffic capture and analysis	Signal characteristic mapping and matching
Scope	Focuses on network behavior and devices	Focuses on spatial signal patterns
Output	Data about devices, networks, and traffic	Estimated location or spatial context

In summary, Wi-Fi monitoring observes and analyzes Wi-Fi traffic for network insights, while Wi-Fi fingerprinting leverages signal characteristics to provide location-based information.

5.3 Radio-based Localisation Techniques

5.3.1 Received Signal Strength (RSS)

RSSI values are Received Signal Strength (RSS) values averaged over a certain sampling period and are measured as a perceived power value P_r with the units of decibels. Using the physical property of **attenuation**, the gradual loss of intensity over a spatial propagation, RSS can be used to determine a distance d to an emitting source, as the output power value P_T is known. RSSI localisation uses trilateration with at least 3 transmitters.

$$P_r \propto P_T \frac{G_T G_R}{4\pi d^p}$$

Attenuation is characterised by a path loss exponent p which in free space is represented by a literature value $p = 2$, it is however heterogeneous in an indoor space. P_T is the transmitted power by the at the emitter. In theory, by calculating multiple perceived distances d from emitters to various receivers, position can be calculated via multi-lateration. This however this is subject to multi-path influences, introducing error.

Advantages:

- Simple and cost-efficient due to low hardware requirements

Disadvantages:

- Poor localization accuracy, especially in non-Line-of-Sight situations
- Additional signal attenuation from transmission through walls, obstacles, and multipath
- Potentially high fluctuation over time

5.3.2 Channel State Information (CSI)

- Captures amplitude and phase response across different frequencies
- Higher granularity than RSSI, gives more information to reduce multipath and provide more consistent measurements
- Overall higher localisation accuracy than RSSI

5.3.3 Fingerprinting/Scene Analysis

- Discrete estimation of user location based on grid
 - Delicate balance between size of grid (and corresponding resolution), and the signal variation error
 - Eg: impossible to estimate the correct point if the difference in RSSI values between each grid cell is less than the uncertainty range of the signal strength
- Offline phase: collect RSSI/CSI measurements to form a fingerprint map
- Online phase: compare real-time measurements with fingerprint map using algorithms:
 - Probabilistic method: use histogram/kernel to calculate probability of receiver being in x grid of fingerprint map
 - Artificial Neural Network: train a neural network (eg: Multi-Layer Perceptron) and provide real-time input with weights
 - k-Nearest Neighbours: average nearest measurement patches to estimate the location
 - Support Vector Machine (SVM): machine learning method applicable to indoor localisation

5.3.4 Angle of Arrival (AoA)

- Receiver's antennae array estimates the angle of transmitted signal by the time difference of arrival at individual parts of the array
- Advantages:
 - Receiver location can be estimated with just 3 transmitters in a 3D environment
 - Accurate estimation when transmitter-receiver distances are small
- Disadvantages:
 - Slight error in angle calculation translates to huge error in position determination
 - * Thus not accurate at larger transmitter-receiver distances
 - Line of Sight for angle calculation is hard to obtain due to indoor multipath effects

5.3.5 Time of Flight (ToF)

- Calculate physical transmitter-receiver distance using signal propagation time and speed of light
- Trilateration is used similar to RSSI to estimate position
- Requires strict time synchronisation between transmitter and receiver, such that timestamps may also be included in the signal
- Advantages:
 - High sampling rate and large signal bandwidth increases the resolution of the position estimation
- Disadvantages:
 - Highly sensitive to obstacles as they deflect the signals, resulting in a longer propagation time

5.3.6 Time of Arrival (ToA)

This principle measures the absolute travel time of a signal from transmitter to receiver. A **euclidian distance** can be derived using the wave speed.

5.3.7 Time Difference of Arrival (TDoA)

- Create hyperbolas between each pair of transmitters
- Intersection point of all hyperbolas represents the position estimate
 - I.e: solve the system of hyperbola equations for X,Y,Z
- Difference between TDoA and ToF
 - ToF uses absolute signal propagation time between each transmitter and receiver
 - TDoA uses the difference between these propagation times
 - * Synchronisation is thus only necessary between transmitters (receiver error is common between transmitter pairs)

5.3.8 Return Time of Flight (RToF)

- Measures the round-trip signal propagation time from transmitter to receiver, and response signal from receiver to transmitter
- Difference between RToF and ToF
 - Only moderate synchronisation between transmitter and receiver is required
 - All factors affecting ToF are worse in RToF due to twice the distance
 - Time delay in transmitting response signal (depending on the receiver's electronics) adds to position estimation error

5.3.9 Phase of Arrival (PoA)

- Measures phase difference from transmission to reception
- Typically used in conjunction with other methods (RSSI, ToF, TDoA) to enhance localisation accuracy
- High accuracy is only achievable with direct line-of-sight

5.3.10 Angle of Arrival (AOA)

This methodology aims to obtain the direction of an incident incoming wave signal. For this, directionally sensitive antennas are needed. By having sensors that are responsive to a specific direction in space, vector addition can be used to evaluate an angle of incidence (arrival).

5.3.11 Path-Loss

The ITU Model for Indoor Attenuation takes into account how radio waves propagate indoors. It models the path loss L , relative to the distance d between emitter and receiver.

$$L = 20 \log f + p \log d + c(k, f) - 28$$

Here f is the radio frequency and c is an empirical floor loss penetration factor. k is the number of floors between transmitter and receiver.

5.3.12 Fine Timing Measurement (FTM)

5.3.13 Radio Frequency Identification (RFID)

A Radio Frequency Identification system consists of an antenna which reads nearby active transceivers (transmit & receive) and passive tags on the radio wave part of the EM-spectrum. These transmit a unique tag ID. To position these tags in space the Cell of Origin (CoO) principle is used. A combination of ToA and AoA is used to pinpoint a location of an RFID tag.

Active RFID sensors are equipped with batteries which interrogate active radio transceivers. They are more costly than their passive opponent but offer long detection ranges of 30m or more. Location detection accuracies on the meter level have been proven possible.

Passive receivers rely on inductive coupling of EM waves and do not require batteries. These rely on receiving enough information by a scanner in order to transmit codes back. They are less costly, but only function in ranges smaller than 2 meters as signals need to travel two ways.

Applications of passive scanners include the identification of buried pipes as well as inventory control.

Using RFID scanners for indoor locations implies either having permanent active RFID sensors casting information or scanners suitable to detect passive ones. Both require large capital cost of installation.

5.3.14 Comparison

Technology	Range	Advantages	Disadvantages
Wifi 802.11 n/ac/ad	35m	Widely available High accuracy Existing infrastructure	Sensitive to noise Complex processing algorithms needed
Wifi 802.11 ah (new)	1km	High coverage range Low power consumption	New technology, not yet tested for indoor environments
Ultrawide Band (UWB)	10-20m	Immune to interference High accuracy	Short Range Additional hardware High cost
Radio Frequency Identification Device (RFID)	200m	Low power consumption High coverage range	Low accuracy
Bluetooth	100m	High throughput High coverage range Low power consumption	Low accuracy Sensitive to noise
Ultrasound	<20m	Less absorption from obstacles	High dependence on sensor placement
Acoustics	<2m	High accuracy	Niche applications Sensitive to sound pollution Additional hardware/anchor points
Visible Light (LEDs)	1.4km	Widely available High potential (with AoA technique) Multipath-free	Requires Line-of-Sight Higher power consumption

5.4 Hybrid and Other Techniques

5.4.1 Meshlium

Meshlium is a device that uses WiFi and Bluetooth scanners to detect other devices, which can be used for a range of applications/research (e.g. Vehicle Traffic Detection).

WiFi and Bluetooth radios (of devices) periodically send out messages, containing:

- MAC address of wireless interface
- Strength of the signal (RSSI)
- Vendor of the smartphone
- WiFi Access Point and Bluetooth friendly name
- Class of Device (CoD) (only when Bluetooth)

MAC address randomization: for privacy randomized MAC address, reverts to “factory” MAC address when connected to WiFi

Adaptive Frequency Hopping (AFH): algorithm that enables Bluetooth radio to dynamically identify channels already in use and avoid them

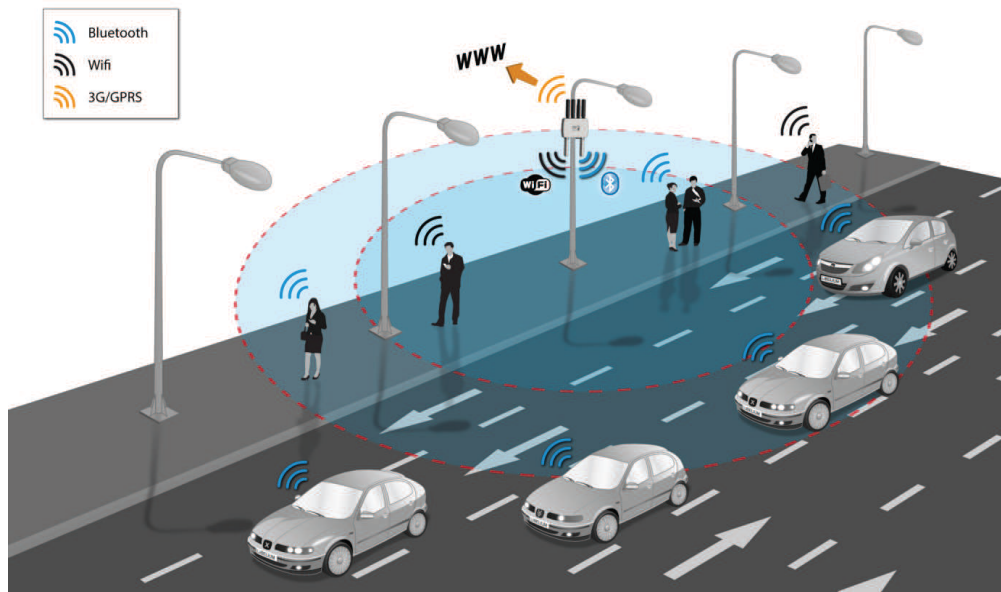


Figure 24: Meshlium Summary

5.4.2 Visual Based Indoor Localisation

- Image processing module: interpret visual data
 - Object recognition in images also help to contextualise the location
 - * Eg: stove, fridge and sink indicate the place is likely a kitchen
 - Local feature descriptors: detect local interest points in an image, describe and store them as words
 - * Bag-of-words model then compares the collection of word descriptors with the map (trained with places also described by bag-of-words) to match a place
 - * Local features do not take the overall geometry into account
 - Thus pose-invariant: place is recognizable regardless of the position/orientation of the source image
 - However, adding geometric information improves robustness of place matching
 - Global feature descriptors: create a fingerprint of a location based on detected features
 - * Uses color histograms, feature detection (edges, corners, color patches)
 - * These features are ordered from 0° to 360° into a fingerprint, using omni-directional cameras during training phase
 - * Assumes the input live data is at similar height/location of the training data
 - Generally, combining both local and global descriptors provides the best results
- Map: maintains a representation of knowledge of the world
 - Usually a relational (topological/cognitive) map rather than absolute/geometric positioning
 - Consists of bounded places
 - * Place signature: set of visual information that distinguishes it from other places
 - * Gateway: physical boundaries of a place, where the physical appearances changes significantly
 - Methods:
 - * Pure image retrieval: matching based only on image similarities, no position information is required/given
 - * Pure topological map: stores relative positions of places, no metric information stored
 - Speeds up searching as indexing is possible
 - * Topological-metric / Topometric map: enhance topological maps with direction and/or distance
 - Appearance-based option: metric information only between places, not within places
 - Sparse landmark option: metric information extracted from depth values between key landmarks inside the image
 - Dense occupancy grid option: same as sparse landmark but for more feature points, more GPU/memory-intensive
- Belief generation: combines information from above components to make decision on place familiarity

- Bag-of-words model: TF-IDF scoring (term frequency - inverse document frequency)
 - * Each visual word in image is scored by frequency of it appearing in image, against how common the word is across all images
- Voting scheme: use multiple data streams to vote confidence of matching
 - * Eg: multiple color bands that give unanimous voting and confidence value $>$ threshold
- Artificial neural network: Continuous attractor network (CAN)
 - * Mimic neural network of a rat hippocampus using local excitation and global inhibition layers

Changing Environments

- Image processing module:
 - Invariant methods: focus on features that are invariant despite changing environments
 - * Eg: edges and corners remain prominent despite lighting changes
 - * This is also true for convolutional neural networks: mid-level features are robust to changes in the environment
 - * Alternatively, use training images that are as ‘change-invariant’ as possible, or pre-process live data to reduce changes
 - Learning methods: define the relationship between how a place can appear across different times
 - * Eg: use pairs of images between two different seasons, or day and night
- Map: how to deal with different representations of the same place
 - Remember and forget: Balance between new observations (that may be fleeting/inconsequential) and overwriting obsolete information
 - Multiple representations of the same environment: to capture cyclic/regular changes in environment
 - * Eg: seasonal changes are cyclic
 - * Store information of the same place (or whole-map level) at different instances of the required timescale

5.5 IndoorGML

Indoor location technologies are a diverse set of technologies that use different physical phenomena and processing techniques to gather and present information. All information has to come together for visualisation purposes, usually in the shape of a map. Compared to a visualisation of the outdoor environment, the specifications for indoor maps are not as easily defined. Therefore a standard, the OGC’s IndoorGML was developed, serving as a standard data model.

Requirements

The required features of a map arise from the constraints the environments present. Other than for an outdoor space, the euclidean distance does not represent the actual distance from point a to b as doors, walls and ceilings might be in the way. Another difference is the purpose of the map. One might divide it up into rooms and hallways, however other representations like public and private areas is possible as well. This requires contextual information of the space, which might be more interesting than the 3D spatial extent of a room.

Rooms, also called cells, are therefore along with their 2- or 3D spatial extent given a set of attributes, such as a classification, usage etc.

5.5.1 Basic Concepts

The OGC published a set of requirements for indoor maps that include but are not limited to:

- Reflecting properties of indoor space
- Cellular space model
- Minimal set of specifications
- Interoperability with other standards
- Extensibility

The OGC’s model is based on the cellular space model. A cellular space is defined as a set of non-overlapping cells, where each cell has an identifier and the union of cells is a subset of the entire indoor space. This is integrated by four main concepts:

- Cell geometry
- Topology between cells
- Cell semantics
- Multilayered space model

5.5.2 Cell Geometry

A 2D surface or 3D solid, defined by one of 3 options:

- No spatial information, only topology
- Spatial and topology
- Only topology, with reference to spatial data sets

5.5.3 Topology between cells

Important for navigation is the spatial relationship between cells. The connection of cells via doors is given via a dual topological graph. The nodes represent rooms, the edges represent doors. Several other attributes may be derived from these like accessibility or travel time between cells.

5.5.4 Cell Semantics

Given that all spaces have different functions, the semantics of cells need to be specified. This includes different applications like indoor facility management. It may also be useful to include a cell boundary as a semantic.

5.5.5 Multilayered Space Model

A space can and needs to be interpreted by different specifications of usage. Therefore a mechanism to include multiple layers for the same space must exist, called interpretations. Each one corresponds to a cellular space layer with its own geometric and topological properties. For example, the topology of a space might be different from the walking and wheelchair interpretation as they are partitioned differently.

6 Location awareness and privacy

6.1 Position, Location, Place and Area

Aspects of addressing space:

- **Reference:** relative (with reference to space or other objects) or absolute (agreed to by general consensus)
- **Specificity** and **Uncertainty:** the extent of the addressable space
- **Scope:** placement at different scales
- **Context:** with or without context

	Position	Location	Place	Area
Reference	Absolute (e.g. coordinate system)	Absolute (e.g. room number)	Relative, placement in a room (inside)	Relative, placement in an aggregation of rooms
Specificity/ Uncertainty	Depends on the device providing the position	Certain, defined by the physical borders (walls)	Uncertain, defined by the functional space of an object (e.g. desk)	Uncertain, defined by a more general notations (floors, parts of building)
Scope	Defined by a reference frame	Contains places	Contained in locations	Contains locations
Context	No context	Context	Context	Context
Example	"I am at 28.2314° - 33.4577°"	"I am in the living room"	"I am at the photocopier"	"I am on the second floor"

Four concepts of placement:

- **Position:** pin-point placements
- **Location:** smallest physically defined space in a building
- **Place:** placement of particular object and the uncertain (functional) space around it
- **Area:** generalised space or sub-space, containing multiple addressable locations

Framework modelling indoor space composed of:

- **Agents:** entities that navigate space, access resources and perform activities
- **Resources**
- **Space:** entirety of the enclosed environment to be navigated
- **Sub-spaces**
 - *Inert spaces:* inaccessible by agents
 - *Free spaces:*
 - * Allow agents to move through them
 - * Contain resources
 - * Host activities
- **Modifiers:**
 - Can be applied to sub-spaces, agents and resources
 - Define the environment of a sub-space, a sub-space can be encumbered by multiple modifiers
- **Activities**

Network models: graph structure $G(V, E)$ representing indoor space

- Nodes V : subdivisions
- Edges E : topological relationship between nodes

6.2 Personal Data Protection in the European Union

6.2.1 Data Processing Terminology

6.2.1.1 Personal Data Under EU law, personal data is defined in the Article 4 of the General Data Protection Regulation (GDPR) as:

[A]ny information relating to an **identified or identifiable natural person** (“data subject”); an identifiable natural person is one who can be identified, **directly or indirectly**, in particular by reference to an identifier such as a *name*, an *identification number*, *location data*, an *online identifier*, or to *one or more factors* specific to the physical, physiological, genetic, mental, economic, cultural, or social identity of that natural person

GDPR, Article 4(1)

A person who’s data is being processed is a ‘**data subject**’.

Information that can be used to identify a person includes:

- Name
- Identification number
- Location data
- Online identifier
- Vehicle registration number
- Physical characteristics
- Genetic data
- Cultural identity

The **metadata** should also be considered, as it sometimes contains even more information. For example, the **metadata of a picture** taken with a smartphone can contain the GPS coordinates, the date and time, the author, the camera model and the settings of the camera.

The concept of identifiability is explained by the Recital 26 of the GDPR. This approach is called the **risk-based approach**:

To determine whether a natural person is **identifiable**, account should be taken of all the means that are **reasonably likely** to be used, such as detection, by the controller or another person, to identify the natural person directly or indirectly. To determine whether the means are reasonably likely to be used to identify the natural person, account should be taken of all objective factors, such as the **cost and time required** for identification, taking into account the **technology available at the time of processing** and **technological developments**

GDPR, Recital 26

The form that personal data takes is not relevant to the laws that govern its usage. CCTV footage, recorded audio, pictures, DNA samples and digital communications are all examples of personal data.

All in all, this problem of distinguishing personal data from non-personal data must be handled as a **dynamic problem**. The controller must continuously **monitor the technological advancements and the capabilities of other actors** to adopt the right measures in due time.

6.2.1.2 Anonymisation Data can be kept in a form that allows for identification **no longer than is necessary for the purposes** for which the data is being processed. After personal data has served its purpose it needs to either be **erased** or **anonymised**. Data is anonymised when all identifying elements are removed.

Data that has been anonymised properly is no longer considered personal data and therefore data protection legislation no longer applies.

However, it was shown by many studies that it is possible to identify an individual through the combination of various anonymised datasets. This process is called **re-identification**.

Therefore, pretending to achieve **anonymisation that is permanent as erasure is utopic**. But it is still better than leaving the data in its initial state, reducing the risk to its lowest possible level.

6.2.1.3 Pseudonymisation In Article 4 of the GDPR, the concept of **pseudonymisation** is defined as:

[P]rocessing of personal data in such a manner that the personal data can no longer be attributed to a **specific data subject** without the use of **additional information**, provided that such additional information is **kept separately** and is subject to **technical and organisational measures** to ensure that the personal data are not attributed to an identified or identifiable natural person

GDPR, Article 4(5)

6.2.1.4 Data Processing Data processing covers a large number of possible actions. Examples include:

- collection
- organisation
- structuring
- storage
- alteration
- retrieval
- usage
- disclosure
- restriction
- erasure

Automated and non-automated processes both count as data processing.

6.2.1.5 Users of Personal Data There are two types of entities that handle personal data: **controllers** and **processors**. A controller is a natural or legal person that determines the purpose and means of processing. A processor is a natural or legal person who processes the data on behalf of the controller. A controller oversees and controls the processing, as well as being responsible and legally liable.

6.2.2 Lawfulness, Fairness and Transparency of Processing Principles

6.2.2.1 Lawfulness of Processing Lawful processing of personal data requires the **consent of the data subject** or **another legitimate reason**. The other five reasons are:

1. When processing personal data is necessary for performance of a **contract**.
2. For the performance of a task by a **public authority**.
3. For compliance with a **legal obligation**.
4. For the purpose of the **legitimate interests** of the controller or third parties.
5. Or if necessary to protect the **vital interests of the data subject**.

6.2.2.1.1 Consent Controllers have a duty to keep a verifiable record of any consent received. Consent can be **withdrawn at any time**. The four characteristics of consent are:

1. **Free:** Consent must be freely given.
2. **Informed:** The data subject must have sufficient information before making a decision.
3. **Specific:** For consent to be valid it must also be specific to the processing purpose.
4. **Unambiguous:** There should be no reasonable doubt that the data subject wanted to express their agreement to the processing of their data.

6.2.2.2 Fairness of Processing Data subjects should **be notified** by controllers that they are processing their data in a lawful and transparent manner, and should be able to demonstrate that they are doing so.

6.2.2.3 Transparency of Processing Controllers are obligated to take appropriate measures to ensure that data subjects remain **informed** about how their data is being used.

6.2.3 Data Processing Principles

6.2.3.1 The Principle of Purpose Limitation Data cannot be processed further in a way that is **not compatible with the original purpose**, although exceptions are possible if the new purpose is either:

- **Archiving** purposes in the public interest.
- **Scientific** or **historical** research.
- **Statistical** purposes.

6.2.3.2 The Data Minimisation Principle Processing of personal data must be **limited** to what is **necessary** to fulfil a legitimate purpose.

6.2.3.3 The Data Accuracy Principle A controller holding personal data is not allowed to process said data without ensuring with reasonable certainty that the data are **correct** and **up to date**.

6.2.3.4 The Storage Limitation Principle Data must be **deleted or anonymised** as soon as they are **no longer needed** for the purposes for which they were collected.

6.2.3.5 The Data Security Principle Controllers of personal data are required to implement appropriate **technical or organisational measures** when processing data. How appropriate a security measure is depends on the context and is determined on a **case-by-case basis** and should be regularly reviewed.

6.2.3.6 The Accountability Principle Controllers and processors are required to **actively and continuously** implement measures to promote and safeguard **data protection** in their processing activities.

6.2.4 Specific to Location Data

6.2.4.1 Sources of Location Data Location data comes from a variety of sources, including:

- **GNSS**
- **Wi-Fi**
- **Cell Phone Tracking**
- **Bluetooth Beacons transmitters**

The **diversity of sensors** inside mobile devices (microphone, camera, infrared, GPS, Bluetooth, accelerometer, Wi-Fi, fingerprint sensor, etc.) and the widespread use of various **mobile apps** make it easy to collect and combine a wide range of data. This data can then be combined with other data sources to **infer private information** about the user.

All this data is accessed by apps through APIs provided by the **operating system** (OS), which also exploits the data for its own purposes.

6.2.4.2 European Framework For location data, besides the GDPR, the European legal framework also encompasses the **e-Privacy Directive**, which establishes rules to ensure privacy and personal data protection in the electronic communications sector.

The Article 2(c) of the e-Privacy Directive defines **location data** as:

[A]ny data processed in an electronic communications network or by an electronic communications service, indicating the **geographic position** of the terminal equipment of a user of a publicly available electronic communications service

e-Privacy Directive, Article 2(c)

The Recital 14 the specifies that such data:

[M]ay refer to the *latitude, longitude and altitude* to the user's terminal equipment, to the *direction of travel*, to the *level of accuracy* of the location information, to the *identification* of the network cell in which the terminal equipment is located at a certain point in time and to the *time* the location information was recorded

e-Privacy Directive, Recital 14

The Article 9 of the GDPR also establishes **special categories of personal data** which are particularly sensitive, such as racial or ethnic origin, political opinions, religious or philosophical beliefs, genetic biometric and health data or data concerning a natural person's sex life or sexual orientation. Location data may help infer such data, making it particularly sensitive.

6.2.4.3 Some Selected Cases

6.2.4.3.1 Location of Employees According to WP29, the use of **geolocation of employees** can find legal basis in the **legitimate interest** of the employer, who is the data controller. However, the employer must be able to demonstrate the **necessity** of the processing and the **balance of the interests** of the employer and the employees. The employer must also inform the employees about the processing of their location data. In its Opinion 8/2001, WP29 states that consent can hardly be a legal basis due to the **dependency of the employee**, making the consent **not freely given**.

In Italy, remote control of employees is limited to **specific cases** and **specific conditions** (organisational and production needs, workplace safety, protection of company assets) and must be negotiated with **union representatives** first.

In France, it is only allowed for control services related to the vehicle usage, ensuring the security of employees and goods and checking working hours. It requires a prior **Data Protection Impact Assessment (DPIA)**.

6.2.4.3.2 Smart Vehicles Smart vehicles are equipped with a wide range of sensors and communication systems, which can collect a wide range of data, including location data. This location data is particularly sensitive, as it can reveal the **habits and preferences** of the driver. The **data controller** — which can be the vehicle producer, the equipment manufacturers or the service providers — shall **make the data subject aware** of how the data is processed, i.e. the frequency of collection, the possibility to shut down the tracking system and the third parties that can access the data.

The collection of location data shall be **proportionate** to the purposes by modulating the *frequency* and the *precision*. The purpose also influences the length of the data retention (data minimisation principle). For security reasons, personal data should also be **processed internally** as far as possible, and only sent to third parties when absolutely necessary.

6.2.4.3.3 Contact Tracing **Digital Contact Tracing (DCT)** apps use tracking technologies to monitor the simultaneous presence of individuals in the same place. There are two main ways to implement DCT:

- Using **proximity data**, usually with *Bluetooth Low Energy (BLE)* beacons. The absolute position is unknown and data is stored locally on the device unless a user is tested positive. This approach was used by Trace Together in Singapore.
- Using **location data**, usually with GNSS. The absolute position is known, and the data is stored on a central server. This approach was used by WeChat and Alipay in China.

In Europe, the European Commission and the European Data Protection Board (EDPB) have expressed a preference for BLE for privacy reasons. The EDPB also gave criteria for the adoption of DCT apps:

- **Voluntary** use
- **DPIA** before development
- Predilection for **proximity** data
- Disclosure of information on who the infected has been in close contact with

- Data **minimisation** and data **protection** by design
- **Encrypted identifiers** generated by BLE
- **Anonymity** of third users involved